UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY III
COURSE CODE	:	BNJ 22403
PROGRAMME CODE	:	BNG/BNM
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER **ALL** QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 Given $z = f(x, y)$ where $x^2 \sin(2y - 5z) + x^2 y^2 = xe^z + 3x$. Find $\frac{\partial z}{\partial x}$ using implicit differentiation.

(8 marks)

Q2 Supposed that $p = r^2 - r\tan\theta$ with $r = \sqrt{s}$ and $\theta = \pi s$.

(a) Find $\frac{dp}{ds}$ in terms of s using chain rule.

(5 marks)

(b) Hence, evaluate $\frac{dp}{ds}$ when $s = \frac{1}{4}$.

(2 marks)

Q3 Compute the volume of a solid G in the first octant which bounded by the plane $4x + y + 2z = 4$ using triple integrals.

(7 marks)

Q4 Given a solid G enclosed by a paraboloid $z = 5 + x^2 + y^2$ and plane $z = 9$.

(a) Sketch the graph of the solid G.

(3 marks)

(b) Hence, evaluate the volume of solid G using double integrals.

(6 marks)

Q5 Given a nonlinear equation, $f(x) = x^3 + \cos x$.

(a) Choose an interval from the following given intervals that contains the root for $f(x) = 0$ using Intermediate Value Theorem (IVT).

$$-2 \leq x \leq -1.75 \quad , \quad -1.5 \leq x \leq -1.25 \quad , \quad -1 \leq x \leq -0.75.$$

(3 marks)

(b) Hence, estimate the root of $f(x) = 0$ using secant method. Iterate until $|f(x_i)| \leq 0.005$.

(6 marks)

Q6 A biologist has placed three strains of bacteria (denoted I, II and III) in a test tube, where they will feed on three different food sources (A, B and C) every day. Each bacteria consumes a certain number of units of each food per day. The number of units consumes and the number of food sources are shown in the **Table Q6**.

Table Q6 Bacteria Strains and the Number of Food Sources

	Bacteria Strain I	Bacteria Strain II	Bacteria Strain III	Number of Food Sources
Food A	4	2	0	350
Food B	0	2	3	500
Food C	5	3	1	600

Let x_1, x_2 and x_3 denoted as Bacteria Strain I, Bacteria Strain II and Bacteria Strain III respectively.

(a) Form a system of linear equations in (augmented matrix form) for the above problem.

(3 marks)

(b) Hence, calculate the number of bacteria of each strain that can coexist in the test tube and consume all of the food by using

(i) Gauss elimination method.

(5 marks)

(ii) Thomas algorithm.

(10 marks)

Q7 The vapour pressure, P (mm Hg) of water is presented as a function of temperature, T (°C) as listed in **Table Q7**.

Table Q7 Vapour pressure of water

T	40	48	56	64	72	80
P	55	a	124	170	b	260

Determine the values of a and b by using Lagrange interpolating polynomial.

(10 marks)

Q8 **Table Q8** represents a set of discrete data.

Table Q8 A set of discrete data

x	1.1	1.3	1.5	1.7	1.9
$g(x)$	0.672	3.527	9.891	22.438	45.146

(a) Compute the value of $g'(1.5)$ by using 2-point backward difference and 3-point forward difference formulas.

(4 marks)

(b) Approximate the value of $g''(1.5)$ by using 5-point difference formula.

(2 marks)

Q9 Given $\int_{2}^{4.7} \frac{e^x - 2x}{x+1}\, dx$.

(a) Calculate the integral by using an appropriate Simpson's rule with $h = 0.3$.

(10 marks)

(b) Calculate the absolute error for answer obtained in **Q9(a)** if the exact solution is 17.295.

(2 marks)

Q10 Given a matrix,

$$A = \begin{pmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{pmatrix}.$$

(a) Find the shifted matrix A, A_{shifted}.

(3 marks)

(b) Hence, compute the smallest eigenvalue, λ_{shifted} for matrix A and its corresponding eigenvector by using the shifted power method, given that the dominant eigenvalue for matrix A is $\lambda_{\text{largest}} = 4.018$. Use initial eigenvector, $\mathbf{v}^{(0)} = (1\ 1\ 1)^T$ and stop the iteration until $|m_{k+1} - m_k| < 0.005$.

(11 marks)

– END OF QUESTIONS –

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Formulas

Multiple Integrals:

Polar coordinates: $x = r\cos\theta,\ y = r\sin\theta,\ x^2 + y^2 = r^2, \quad 0 \le \theta \le 2\pi$

$$\iint_R f(x,y)\ dA = \iint_R f(r,\theta)\, r\ dr\, d\theta$$

Nonlinear equations:

Secant method: $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)},\ i = 01, 2, \ldots$

System of linear equations:

Thomas algorithm:

i	**1**	**2**	**...**	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				

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Formulas

Interpolation:

Lagrange polynomial: $P_n(x)=\sum_{i=0}^{n} L_i(x)f(x_i), i=0,1,2,\ldots,n$ where $L_i(x)=\prod_{\substack{j=0\\ j\neq i}}^{n}\frac{(x-x_j)}{(x_i-x_j)}$

Numerical differentiation and integration:

Differentiation:

First derivative:

2-point backward difference: $f'(x)\approx\frac{f(x)-f(x-h)}{h}$

3-point forward difference : $f'(x)\approx\frac{-f(x+2h)+4f(x+h)-3f(x)}{2h}$

Second derivative:

5-point difference : $f''(x)\approx\frac{-f(x+2h)+16f(x+h)-30f(x)+16f(x-h)-f(x-2h)}{12h^2}$

Integration:

Simpson's $\frac{1}{3}$ rule: $\int_a^b f(x)\ dx\approx\frac{h}{3}\left[f_0+f_n+4\sum_{\substack{i=1\\ i\text{ odd}}}^{n-1} f_i+4\sum_{\substack{i=2\\ i\text{ even}}}^{n-2} f_i\right]$

Simpson's $\frac{3}{8}$ rule: $\int_a^b f(x)\ dx\approx\frac{3}{8}h\left[\begin{matrix} f_0+f_n+3(f_1+f_2+f_4+f_5+\ldots+f_{n-2}+f_{n-1}) \\ +2(f_3+f_6+\ldots+f_{n-3})\end{matrix}\right]$

Eigenvalue

Power Method : $\mathbf{v}^{(k+1)}=\frac{\mathbf{A}\mathbf{v}^{(k)}}{m_{k+1}},\quad k=0,1,2,\ldots$

Shifted Power Method : $\mathbf{A}_{\text{shifted}}=\mathbf{A}-\lambda_{\text{largest}}\mathbf{I},\quad \lambda_{\text{smallest}}=\lambda_{\text{shifted}}+\lambda_{\text{largest}}$