UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ELECTROMAGNETICS TECHNOLOGY
COURSE CODE	:	BNR 20603
PROGRAMME CODE	:	BND/ BNE/ BNF
EXAMINATION DATE	:	JULY/ AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	1.	ANSWER ALL QUESTIONS
	2.	THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.**
	3.	STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **TWELVE (12)** PAGES

PART 1 : OBJECTIVE QUESTIONS (10 QUESTIONS, 20 MARKS)

Q1 (a) Two-point charges $Q_1 = +1$ nC and $Q_2 = +2$ nCaret a distance apart. Which of the following statement is true ?

(a) The force on Q_1 is attractive.
(b) The force on Q_2 is the same in magnitude as that on Q_1.
(c) As the distance between them decreases, the force on Q_1 increases linearly.
(d) The force on Q_2 is along the line joining them.

(2 marks)

(b) Which of the following statements about electric flux density (**D**) in a dielectric medium is true?

i. **D** is proportional to the applied electric field (**E**) and the permittivity of free space (ε_o).
ii. **D** is equal to the applied electric field (**E**) times the permittivity of the medium (ε).
iii. **D** is proportional to the electric field (**E**) and the polarization vector (**P**).
iv. **D** is equal to the applied electric field (**E**) times the relative permittivity of the medium (ε_r).

(a) i & ii & iii
(b) ii & iii
(c) i & iii & iv
(d) i & ii & iv

(2 marks)

(c) A closed Gaussian surface encloses three point charges with the value of +5 nC. +5 nC and -10 nC. What is the electric flux through the surface?

(a) 10 nC
(b) 20 nC
(c) 0 C
(d) Cannot be determined.

(2 marks)

(d) In a conductor, the resistance increases as the temperature increases. What is the primary cause of this behaviour?

i. Increased lattice vibrations.
ii. Frequent collisions between the lattice ions and the electrons.
iii. Reducing the electrons' mobility.
iv. Increased electron-electron collisions.

(a) i & ii & iii
(b) ii & iii & iv
(c) i & ii & iv
(d) All of the above.

(2 marks)

(e) What is the best solution to store the maximum energy in a parallel-plate capacitor with a given battery (voltage source)?

i. The plates far apart.
ii. The plates close together.
iii. Increase the area of each plate.
iv. Reduce the area of each plate.

(a) i & ii, iii
(b) i & ii
(c) ii & iii
(d) ii & iv

(2 marks)

(f) Electric current is generally caused by the motion of electric charges. Which of the following statements are incorrect?

(a) Electric current flowing in a copper wire is an example of convection current.
(b) The resistance of a conductor decreases as the uniform cross-section of the area increases.
(c) The conductivity of conductors and insulators varies with frequency.
(d) The electric susceptibility of dielectric measures the sensitivity of the material to an electric field.

(2 marks)

(g) Maxwell's equations consist of four fundamental equations that govern the behavior of electric and magnetic fields. Which of the following statements wrongly captures the essence of Maxwell's equations?

i. Faraday's Law of Induction: A changing magnetic field induces an electromotive force (EMF) in a conducting loop, resulting in a circulating electric field.
ii. Bio-Savart Law for Magnetism: The net magnetic flux through a closed surface is not always zero, indicating the absence of magnetic dipoles.
iii. Gauss's Law for Electricity: The net electric flux through a closed surface is proportional to the enclosed electric charge.
iv. Ampere's circuit Law: The circulation of the electric field around a closed loop is proportional to the sum of the enclosed current and the time rate of change of electric flux.

(a) ii & iii & iv
(b) ii & iv
(c) i & ii & iiii
(d) All of the above

(2 marks)

(h) Which of the following **Figure Q1(h)** correctly represents the, $\vec{\mathbf{I}}$ and $\vec{\mathbf{H}}$ configurations?

(a) a & b & c
(b) b & e
(c) c & e
(d) a & d

(2 marks)

(i) In a conducting material with a certain conductivity, the skin depth is found to be δ_1 at frequency f_1 and δ_2 at frequency f_2. If $\delta_1 > \delta_2$, which of the following is true regarding f_1 and f_2?
(a) $f_1 > f_2$
(b) $f_1 < f_2$
(c) $f_1 = f_2$
(d) The relationship between f1 and f2 cannot be determined from the given information.

(2 marks)

(j) What is the major factor for determining whether a medium is free space, a lossless dielectric, a lossy dielectric, or a good conductor?
(a) Attenuation constant.
(b) Constitutive parameters σ, ε, μ.
(c) Loss tangent.
(d) Reflection coefficient.

(2 marks)

PART 2 : SUBJECTIVE QUESTIONS (4 QUESTIONS, 80 MARKS)

Q2 (a) Determine the $\boldsymbol{F}_2$ in vacuum, on a point charge $Q_2 = 10^{-6}C$ due to a point charge $Q_1 = 2\times10^{-5}C$ when Q_2 is at the point P_2 (2,4,5) and Q_1 is at point P_1 (0,1,2).

(6 marks)

(b) Given $\rho_v = \begin{cases} \dfrac{10}{r^2} mC/m^3 & 1 < r < 4 \\ 0 & r > 0 \end{cases}$

Analyze the net flux crossing surface $r = 2$ and $r = 6$ m.

(7 marks)

(c) Three point charges $Q_1 = 1\ mC$, $Q_2 = -2\ mC$, and $Q_3 = 3\ mC$ are, respectively, located at $(0, 0, 4)$, $(-2, 5, 1)$, and $(3, -4, 6)$. Determine the potential difference V_{PQ}. The point P is located at $(-1, 1, 2)$ and point Q is located at $(1, 2, 3)$.

(7 marks)

Q3 (a) Briefly explain the differences between convection and conduction current density.

(5 marks)

(b) A homogeneous dielectric ($\varepsilon_r = 2.5$) fills region 1 ($x \le 0$) while region 2 ($x \le 0$) is free space. If $\boldsymbol{D_1} = 12\boldsymbol{a_x} - 10\boldsymbol{a_y} + 4\boldsymbol{a_z}\ nC/m^2$, Analyzes $\boldsymbol{D_2}$ and θ_2.

(8 marks)

(c) Determine the capacitance per unit length of the infinite coaxial cable as shown in **Figure Q3(c)**.

(7 marks)

Q4 (a) A 'C' shape conducting filament carries 10 A current placed on $z = 0$ plane shown in **Figure Q4 (a)**. Evaluate **H** at (0, 0, 2).

(10 marks)

(b) Two infinitely long solid conductors whose cross-section is illustrated in **Figure Q4 (b)**. Both conductors are separated by 8 m. The wire centered at (0, 0, 0) carries a current of 10 A while the other centered at (8 m, 0, 0) carries the return current. Find **H** at (8 m, 4 m, 0).

(10 marks)

Q5 (a) The skin depth phenomenon is essential in understanding the behaviour of electromagnetic waves in conductive materials. Provide a detailed explanation of the skin depth concept and its significance in various practical applications. Your answer should address the basic formulation with the aid of illustration. This should also include the factors that influence the skin depth and their behaviours in good and poor conductors.

(5 marks)

(b) Two conducting bars slide over two stationary rails as shown in **Figure Q5(b)**. If $\mathbf{B} = 0.2\boldsymbol{a_z}$ Wb/m^2, determine the induced emf in the loop thus formed.

(8 marks)

(c) Wave propagation under seawater for submarine communication is a crucial element in establishing reliable communication. Assuming that parameters $\sigma = 4$ S/m, $\varepsilon_r = 80$, $\mu_r = 1$, and $f = 100$ kHz, Calculate

(i) the phase velocity
(ii) the wavelength
(iii) the skin depth

(7 marks)

-END OF QUESTIONS –

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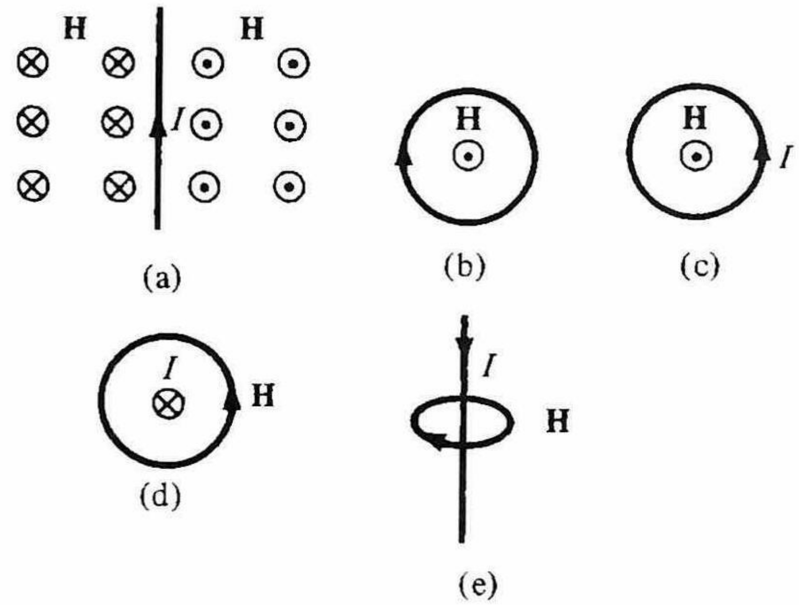

Figure Q1(h)

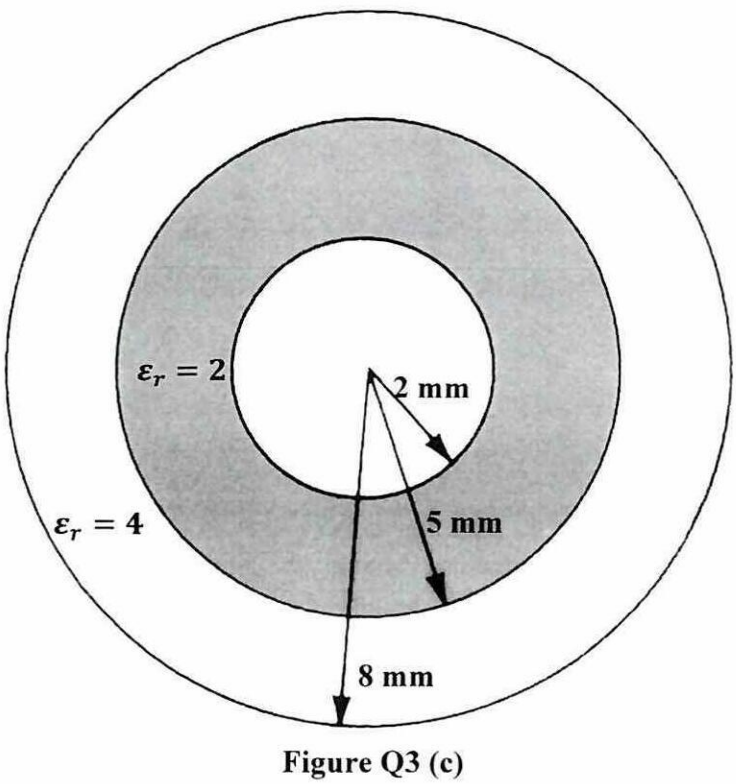

Figure Q3 (c)

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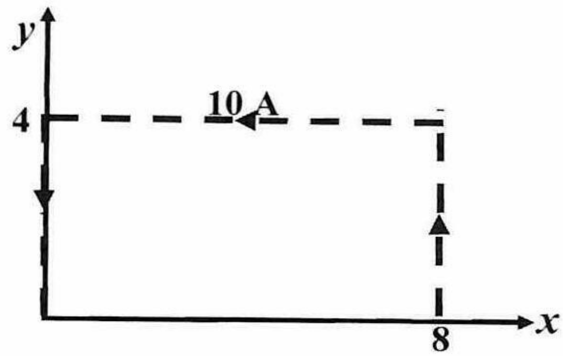

Figure Q4 (a)

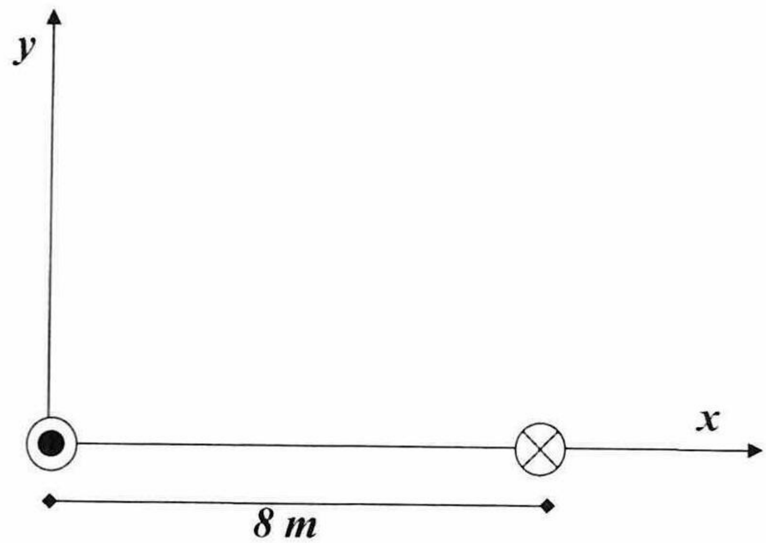

Figure Q4 (b)

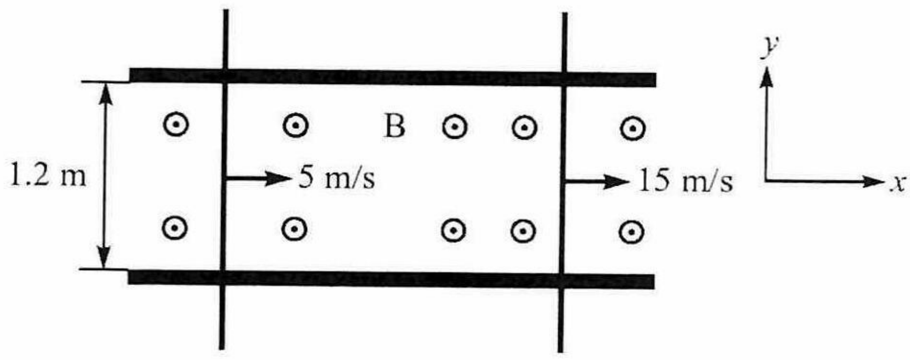

Figure Q5 (b)

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Formula

Gradient of a scalar

Cartesian	Cylindrical	Spherical
$\nabla V = \frac{\partial V}{\partial x}\underset{\sim}{a}_x + \frac{\partial V}{\partial y}\underset{\sim}{a}_y + \frac{\partial V}{\partial z}\underset{\sim}{a}_z$	$\nabla V = \frac{\partial V}{\partial \rho}\underset{\sim}{a}_\rho + \frac{1}{\rho}\frac{\partial V}{\partial \phi}\underset{\sim}{a}_\phi + \frac{\partial V}{\partial z}\underset{\sim}{a}_z$	$\nabla V = \frac{\partial V}{\partial r}\underset{\sim}{a}_r + \frac{1}{r}\frac{\partial V}{\partial y}\underset{\sim}{a}_\theta + \frac{1}{r\sin\theta}\frac{\partial V}{\partial \varphi}\underset{\sim}{a}_\varphi$

Divergence of a vector

Cartesian	Cylindrical	Spherical
$\nabla \bullet \overline{V} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}$	$\nabla \bullet \overline{V} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho V_\rho\right) + \frac{1}{\rho}\frac{\partial V_\phi}{\partial \phi} + \frac{\partial V_z}{\partial z}$	$\nabla \bullet \overline{V} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V_r\right) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}\left(V_\theta \sin\theta\right) + \frac{1}{r\sin\theta}\frac{\partial V_\phi}{\partial \varphi}$

Curl of a vector

Cartesian
$\nabla \times \overline{V} = \left[\frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z}\right]\underset{\sim}{a}_x + \left[\frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x}\right]\underset{\sim}{a}_y + \left[\frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y}\right]\underset{\sim}{a}_z$
Cylindrical
$\nabla \times \overline{V} = \left[\frac{1}{\rho}\frac{\partial V_z}{\partial \phi} - \frac{\partial V_\phi}{\partial z}\right]\underset{\sim}{a}_\rho + \left[\frac{\partial V_\rho}{\partial z} - \frac{\partial V_z}{\partial \rho}\right]\underset{\sim}{a}_\phi + \frac{1}{\rho}\left[\frac{\partial\left(\rho V_\phi\right)}{\partial \rho} - \frac{\partial V_\rho}{\partial \phi}\right]\underset{\sim}{a}_z$
Spherical
$\nabla \times \overline{V} = \frac{1}{r\sin\theta}\left[\frac{\partial\left(V_\phi \sin\theta\right)}{\partial \theta} - \frac{\partial V_\theta}{\partial \phi}\right]\underset{\sim}{a}_r + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial V_r}{\partial \phi} - \frac{\partial\left(rV_\phi\right)}{\partial r}\right]\underset{\sim}{a}_\theta + \frac{1}{r}\left[\frac{\partial\left(rV_\theta\right)}{\partial r} - \frac{\partial V_r}{\partial \theta}\right]\underset{\sim}{a}_\phi$

Laplacian of a scalar

Cartesian
$\nabla^2 V = \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2}$
Cylindrical
$\nabla^2 V = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho\frac{\partial V}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 V}{\partial \phi^2} + \frac{\partial^2 V}{\partial z^2}$
Spherical
$\nabla^2 V = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial V}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial V}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 V}{\partial \phi^2}$

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Differential length $d\bar{l}$

Cartesian	Cylindrical	Spherical
$d\bar{l} = dx\underset{\sim}{a}_x + dy\underset{\sim}{a}_y + dz\underset{\sim}{a}_z$	$d\bar{l} = d\rho\underset{\sim}{a}_\rho + \rho d\phi\underset{\sim}{a}_\phi + dz\underset{\sim}{a}_z$	$d\bar{l} = dr\underset{\sim}{a}_r + rd\theta\underset{\sim}{a}_\theta + r\sin\theta d\phi\underset{\sim}{a}_\phi$

Differential normal area $d\bar{S}$

Cartesian	Cylindrical	Spherical
$d\bar{S} = dydz\underset{\sim}{a}_x$ $dxdz\underset{\sim}{a}_y$ $dxdy\underset{\sim}{a}_z$	$d\bar{S} = \rho d\phi dz\underset{\sim}{a}_\rho$ $d\rho dz\underset{\sim}{a}_\phi$ $\rho d\phi d\rho\underset{\sim}{a}_z$	$d\bar{S} = r^2\sin\theta d\theta d\phi\underset{\sim}{a}_r$ $r\sin\theta drd\phi\underset{\sim}{a}_\theta$ $rdrd\theta\underset{\sim}{a}_\phi$

Differential volume dv

Cartesian	Cylindrical	Spherical
$dv = dxdydz_z$	$dv = \rho d\rho d\phi dz$	$dv = r^2\sin\theta drd\theta d\phi$

Electrostatic

$Q = \int \rho_l dl$	$Q = \int \rho_s dS$	$Q = \int \rho_v dv$
$\vec{F}_{12} = \dfrac{Q_1Q_2}{4\pi\varepsilon r^2}a_{12}$	$\bar{E} = \dfrac{\bar{F}}{Q}$	$\vec{E} = \int \dfrac{\rho_l dl}{4\pi\varepsilon r^2}\underset{\sim}{a}_r$
$\vec{E} = \int \dfrac{\rho_s dS}{4\pi\varepsilon r^2}\underset{\sim}{a}_r$	$\vec{E} = \int \dfrac{\rho_v dv}{4\pi\varepsilon r^2}\underset{\sim}{a}_r$	$\bar{D} = \varepsilon\vec{E} = \dfrac{Q_{enc}}{S}$
$\varepsilon = \varepsilon_o\varepsilon_r$	$\psi_e = \int \bar{D} \bullet d\bar{S}$	$\psi = Q_{enc} = \oint \bar{D}\cdot d\bar{S}$
$\rho_v = \nabla \bullet \bar{D}$	$V_{AB} = -\int_A^B \bar{E} \bullet d\bar{l} = \dfrac{W}{Q}$	$V = \dfrac{Q}{4\pi\varepsilon r}$
$V = \int \dfrac{\rho_l dl}{4\pi\varepsilon r}$	$\oint \bar{E}\cdot d\bar{l} = 0$	$\nabla \times \bar{E} = 0$
$\bar{E} = -\nabla V$	$\nabla^2 V = 0$	$\nabla^2 V = -\dfrac{\rho_v}{\varepsilon}$
$R = \dfrac{\ell}{\sigma S} = \dfrac{\rho_c \ell}{S}$	$I = \int \bar{J} \bullet d\bar{S} = JS$	$J = \sigma E = \rho_v u$
$\rho_v = ne$	$\varepsilon_r = 1 + \chi_e$	$\bar{P} = \chi_e\varepsilon_o\bar{E}$
$\rho_s = \bar{D} \bullet \underset{\sim}{a}_n = D_n$	$\rho_{ps} = \bar{P} \bullet \underset{\sim}{a}_n = P_n$	$V = Ed$

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Electrostatic Boundary Condition (Dielectric-Dielectric)

$\overline{E} = \overline{E_n} + \overline{E_t}$	$\overline{D} = \overline{D_n} + \overline{D_t}$	$\tan\theta = \frac{E_t}{E_n} = \frac{D_t}{D_n}$
$E_{1t} = E_{2t}$	$D_{1n} = D_{2n}$	$\frac{\tan\theta_1}{\tan\theta_2} = \frac{\varepsilon_{r1}}{\varepsilon_{r2}}$
$\frac{E_{1n}}{\varepsilon_{r2}} = \frac{E_{2n}}{\varepsilon_{r1}}$	$\frac{D_{1t}}{\varepsilon_1} = \frac{D_{2t}}{\varepsilon_2}$	$w_e = \frac{1}{2}\varepsilon\left\|\overline{E}\right\|^2$ $W_E = \int w_e dv$

Electrostatic Boundary Condition (Conductor-Dielectric)

$\overline{E} = 0 \quad \rho_v = 0$	$D_t = \varepsilon_o\varepsilon_r E_t = 0$	$D_n = \varepsilon_o\varepsilon_r E_n = \rho_s$

Electrostatic Boundary Condition (Conductor-Free Space)

$\varepsilon_r = 1$	$D_t = \varepsilon_o E_t = 0$	$D_n = \varepsilon_o E_n = \rho_s$

Electrostatic Boundary-Value Problems (Resistance and Capacitance)

$R = \frac{V}{I} = \frac{\int \tilde{E}.d\tilde{l}}{\oint \sigma\tilde{E}.d\tilde{S}}$	$C = \frac{Q}{V} = \frac{\varepsilon\oint \sigma\tilde{E}.d\tilde{S}}{\int \tilde{E}.d\tilde{l}}$	Parallel-plate Capacitor $C = \frac{Q}{V} = \frac{\varepsilon S}{d} \quad R = \frac{d}{\sigma S}$
Parallel-plate Capacitor $W_E = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$	Coaxial/Cylindrical Capacitor $C = \frac{Q}{V} = \frac{2\pi\varepsilon L}{\ln\frac{b}{a}} \quad R = \frac{\ln\frac{b}{a}}{2\pi\sigma L}$	Spherical Capacitor $C = \frac{Q}{V} = \frac{4\pi\varepsilon}{\frac{1}{a}-\frac{1}{b}} \quad R = \frac{\frac{1}{a}-\frac{1}{b}}{4\pi\sigma}$
Spherical Capacitor $RC = \frac{\varepsilon}{\sigma}$		

TERBUKA

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Magnetostatics

$d\overline{H} = \frac{Id\ell \times \overline{R}}{4\pi R^3}$	$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J}dv$	$\nabla \times \overline{H} = \overline{J}$
$H = \frac{I}{4\pi\rho}(\cos\alpha_2 - \cos\alpha_1)\underline{a}_\phi$	$I_{enc} = \oint \tilde{H}.d\tilde{l} = \int \tilde{J}_s d\tilde{S}$	$\psi_m = \int \overline{B}.d\overline{S} = BS = \int_{enc} \overline{B}.d\overline{S}$
$\psi_m = \oint \bar{B}.d\bar{S} = 0$	$\psi_m = \oint \bar{A}.d\bar{l}$	$\nabla.\overline{B} = 0$
$\overline{B} = \mu\overline{H}$ $\lvert\overline{B}\rvert = \frac{\mu_o NI}{\ell}$	$\overline{B} = \nabla \times \overline{A}$	$\overline{A} = \int \frac{\mu_o Id\ell}{4\pi R}$
$\nabla^2\overline{A} = -\mu_o\overline{J}$	$\overline{F} = Q(\overline{E} + \overline{u} \times \overline{B}) = m\frac{d\overline{u}}{dt}$	$d\overline{F} = Id\overline{\ell} \times \overline{B}$
$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$	$\overline{m} = IS\underline{a}_n$	$H = \frac{I}{2\pi\rho}\underline{a}_\phi \quad (l = \infty)$

Time Varying & Faraday Law

$V_{emf} = -\frac{\partial\psi}{\partial t} = \frac{-\partial B}{\partial t}S = IR$	$V_{emf} = -\int \frac{-\partial B}{\partial t} \bullet dS$	$V_{emf} = \int\left(\overline{u} \times \overline{B}\right) \bullet d\overline{\ell}$
$I_d = \int J_d \bullet d\overline{S}$	$J_d = \frac{\partial\overline{D}}{\partial t}$	$\oint \bar{E}.d\bar{l} = -\frac{d}{dt}\int \bar{B}.dS = V_1 - V_2$

Wave Propagation

$\gamma = \alpha + j\beta$	$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$	$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$
Tangent Loss $u = \tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$	$\tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\overline{J}_s}{\overline{J}_{ds}}$	$\lvert\eta\rvert = \frac{\sqrt{\frac{\mu}{\varepsilon}}}{\sqrt[4]{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2}}$
Complex Permittivity $\varepsilon_c = \varepsilon' - j\varepsilon'' = \varepsilon - j\frac{\sigma}{\omega}$		

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Derivatives

$\frac{d}{dx}\log_a U = \frac{\log_a e}{U}\frac{dU}{dx}$	$\frac{d}{dx}\ln U = \frac{1}{U}\frac{dU}{dx}$	$\frac{d}{dx}a^u = d^u \ln a \frac{dU}{dx}$
$\frac{d}{dx}e^u = e^u \frac{dU}{dx}$	$\frac{d}{dx}\sin U = \cos U \frac{dU}{dx}$	$\frac{d}{dx}\cos U = -\sin U \frac{dU}{dx}$
$\frac{d}{dx}\tan U = \sec^2 U \frac{dU}{dx}$	$\frac{d}{dx}\sinh U = \cosh U \frac{dU}{dx}$	$\frac{d}{dx}\cosh U = \sinh U \frac{dU}{dx}$
$\frac{d}{dx}\tanh U = \text{sech}^2 U \frac{dU}{dx}$		

Indefinite Integrals

$\int e^U dU = e^U + C$	$\int e^{ax} dU = \frac{1}{a}e^{ax}$	$\int xe^{ax}dx = \frac{e^{ax}}{a^2}(ax-1)$
$\int x^2 e^{ax}dx = \frac{e^{ax}}{a^3}(a^2x^2 - 2ax + 2)$	$\int \ln x dx = x\ln x - x$	$\int \sin axdx = -\frac{1}{a}\cos ax$

$\int \cos axdx = \frac{1}{a}\sin ax + c$	$\int \tan axdx = \frac{1}{a}\ln\sec ax = -\frac{1}{a}\ln\cos ax$
$\int \sin^2 axdx = \frac{x}{2} - \frac{\sin 2ax}{4a} + c$	$\int \cos^2 axdx = \frac{x}{2} + \frac{\sin 2ax}{4a} + c$
$\int x\cos axdx = \frac{1}{a^2}(\cos ax + ax\sin ax)$	$\int e^{ax}\sin bxdx = \frac{e^{ax}}{a^2+b^2}(a\sin bx - b\cos bx)$
$\int e^{ax}\cos bxdx = \frac{e^{ax}}{a^2+b^2}(a\cos bx + b\sin bx)$	$\int x\sin axdx = \frac{1}{a^2}(\sin ax - ax\cos ax)$
$\int \sec axdx = \frac{1}{a}\ln(\sec ax + \tan ax)$	$\int \sin ax\sin bxdx = \frac{\sin(a-b)x}{2(a-b)} - \frac{\sin(a+b)x}{2(a+b)}$
$\int \sinh axdx = \frac{1}{a}\cosh ax$	$\int \sin ax\cos bxdx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)}$
$\int \cosh axdx = \frac{1}{a}\sinh ax + c$	$\int \cos ax\cos bxdx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)}$
$\int \tanh axdx = \frac{1}{a}\ln\cosh ax$	$\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + C$
$\int \frac{x^2dx}{x^2+a^2} = x - a\tan^{-1}\frac{x}{a} + C$	$\int \frac{xdx}{x^2+a^2} = \frac{1}{2}\ln(x^2+a^2)C$
$\int \frac{xdx}{(x^2+a^2)^{\frac{3}{2}}} = -\frac{1}{\sqrt{x^2+a^2}} + C$	$\int \frac{x^2dx}{(x^2+a^2)^{\frac{3}{2}}} = \ln\left(\frac{\sqrt{x^2+a^2}}{a} + \frac{x}{a}\right) - \frac{x}{\sqrt{x^2+a^2}} + C$
$\int \frac{dx}{(x^2+a^2)^{\frac{3}{2}}} = \frac{x/a^2}{\sqrt{x^2+a^2}} + C$	$\int \frac{dx}{(x^2+a^2)^2} = \frac{1}{2a^2}\left(\frac{x}{x^2+a^2} + \frac{1}{a}\tan^{-1}\frac{x}{a}\right) + C$