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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY II
COURSE CODE	:	BNJ12303/BNP12303/BNT12303/BNR17903/ BNQ10603/BWM12303
PROGRAMMECODE	:	BNJ/ BNR/ BNT/ BNP
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS

2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.**

3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIACLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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Q1 (a) Show that $y = Ae^x + Bxe^x$, where A and B are constants, is the general solution of the differential equation $y'' - 2y' + y = 0$.Hence, find the solution when $y'(0) = 1$ and $y(0) = 1$.

(5 marks)

(b) Use the substitution $z = \cos y$, show that the differential equation

$$\frac{dy}{dx} - \frac{\cot y}{x} = e^{3x} \csc y$$

can reduce to

$$\frac{dz}{dx} + \frac{z}{x} = -e^{3x}.$$

Hence, solve the original equation when $y(1) = 0$.

(10 marks)

(c) (i) Show that the equation $(3x^2 + 2y + 1)dx + (2x + 6y^2 + 2)dy = 0$

is an exact equation.

(3 marks)

(ii) Then, determine the general solution from the given differential equation.

(7 marks)

Q2 (a) Determine which of the following equation is linear. If linear, determine whether or not the equation is homogeneous and state its coefficient form.

(i) $\dfrac{d^2y}{dx^2} + 10\dfrac{dy}{dx} + 2y = 0$

(2 marks)

(ii) $y\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} - 3y = 3x^2$

(2 marks)

(iii) $x^2\dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} + y = 0$

(2 marks)

(b) (i) Examine the general solution of $12\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} - 3y = 0$.

(3 marks)

(ii) Use the result from **Q2 (b) (i)** to solve $12\dfrac{d^2y}{dx^2} - 5\dfrac{dy}{dx} - 3y = e^x$ if given $y(0) = 2$, $y'(0) = 3$.

(8 marks)

(c) Solve the non-homogeneous equation by using the method variation of parameters

$$y'' + 5y' + 6y = e^{-x}.$$

(8 marks)

Q3 (a) Determine the inverse Laplace transform of the following functions.

(i) $F(s) = \dfrac{e^{-4s}}{(s+2)^3}$

(3 marks)

(ii) $F(s) = \dfrac{s+5}{s^2+6s+9}$

(3 marks)

(iii) $F(s) = \dfrac{s}{s^2-9}$

(3 marks)

(b) (i) By using the convolution theorem, show that

$$L^{-1}\left\{\frac{s^2}{(s^2+a^2)^2}\right\} = \frac{1}{2}\left(\frac{1}{a}\sin at + t\cos at\right).$$

(8 marks)

(ii) Solve the differential equation

$$\frac{d^2y}{dt^2} - \frac{dy}{dx} = e^t.$$

When $y(0) = 0$, $y'(0) = 0$

(8 marks)

Q4 (a) The two-dimensional movement of a particle follows an equation

$$\frac{1}{e^x}\frac{dy}{dx}-4x-1=0.$$

At the initial $x=0$, the y movement of the particle is at a position $y=-3$. Determine the displacement y at $x=0.25$, $x=0.5$, $x=0.75$ and $x=1$ using Second order Taylor series method.

(10 marks)

(b) Solve the boundary value problem

$$x''+4x=\sin t,\quad x(0)=0 \text{ and } x(1)=0$$

in the interval $0\le t\le 1$ using finite difference method by taking $\Delta t=h=0.25$.

(15 marks)

- END OF QUESTIONS

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note :r is the least non-negative integer (r = 0, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

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The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y = uy_1 + vy_2,$$

where $u = -\int \frac{y_2 f(x)}{aW}dx + A,\ v = \int \frac{y_1 f(x)}{aW}dx + B$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1 y_2' - y_2 y_1'$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

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Euler's method: $y(x_{i+1}) = y(x_i) + hy'(x_i)$

Fourth-order Runge-Kutta method

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

where $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}) \; k_4 = hf(x_i + h, y_i + k_3)$$

Finite difference method

$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h}, \qquad y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}.$$