Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	STATISTICS FOR ENGINEERING TECHNOLOGY
COURSE CODE	:	BNJ22502 / BNT22502 / BNP22502 / BNR27002
PROGRAMME CODE	:	BNA/ BNB/ BNC/ BND/ BNE/ BNF/ BNG/ BNM/ BNN
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER **ALL** QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 The life time of a certain brand of battery, in hundreds of hours, is modelled by the continuous random variable X with probability density function f (x) given by:

$$f(x) = \begin{cases} 2(x - x^2), & 0 \leq x \leq 1 \\ 0 \quad\quad\quad , & otherwise \end{cases}$$

(a) Compute the average life time of the battery.

(6 marks)

(b) Calculate its variance.

(8 marks)

(c) Calculate $P(X \geq 0.8)$

(4 marks)

Q2 The number of guests arriving at reception of a busy hotel approximately follows a Poisson distribution with an average of three (3) people every twenty (20) minutes in the afternoon ($P_{3,6} = 0.0838$; $P_{9,6} = 0.8843$).

(a) Compute the probability that no arrival occurring between 4:00 pm and 4:20 pm in a typical afternoon.

(5 marks)

(b) Compute the probability that at most six (6) people will check-in every hour in the afternoon.

(6 marks)

Q3 Researchers assume that the levels of vitamin D in the livers of human populations are normally distributed with healthy population have an average of 50 ng/mL of vitamin D in their blood and sick people have around 10 ng/mL. The variances for the two populations are 19600 and 8100 respectively.

(a) Calculate the probability that a random sample of size 15 from the healthy population and size 10 from the sick population will yield a difference between sample means that is greater than or equal to 90. For your information: $P(Z < -1.96) = 0.0250$, $P(Z < 1.96) = 0.9750$ and $P(Z < 1.09) = 0.8621$.

(9 marks)

(b) Sketch the probability distribution of the scenario above highlighting the probability area being calculated.

(2 marks)

Q4 A random sample of 18 men in Kuala Lumpur found that their average age at first marriage was 26 years old with standard deviation of 5 years. A random sample of 12 women in this city found that their average age at first marriage was 22 years old with the standard deviation of 3 years. Assume that the populations data are normally distributed with equal variances,

(a) Identify the type of case study.

(1 mark)

(b) Construct the 90% confidence interval for the true 'age difference' for first marriage between men and women in the city (t-Table: $t_{0.1,30} = 1.310$, $t_{0.05,30} = 1.697$, $t_{0.1,28} = 1.313$, $t_{0.05,28} = 1.701$).

(10 marks)

(c) Summarize the conclusion for the result obtained in **Q4(b)**.

(1 mark)

Q5 A medical researcher wants to determine the range of variance ratio in the blood pressure between male vs. female COVID-19 patients. The result is summarized in **Table Q5.**

Table Q5 Data samples

Male	Female
Number of samples: 25	Number of samples: 13
Mean: 139.4	Mean: 126.3
Standard deviation: 12.3	Standard deviation: 13.5

(a) Construct 95% confidence interval for the ratio of variance between male COVID-19 patients and female COVID-19 patients (f-Table: $f_{0.05,12,24} = 2.18$; $f_{0.025,12,24} = 2.54$, $f_{0.05,24,12} = 2.51$, $f_{0.025,24,12} = 3.02$).

(10 marks)

(b) Summarize the conclusion for the result obtained in **Q5(a)**.

(1 mark)

Q6 A researcher believes that the tissue cultures in her lab are significantly denser than normal. The researcher takes a random sample of 40 specimens of tissue from her lab and finds the average weight is 5 grams per cubic centimetres. Textbook claims that such tissues should weigh 4.7 grams per cubic centimetres with a standard deviation of 0.47 grams. Examine whether there is any evidence to support the researcher's claim at 0.05 level of significance using the following guided questions. Note: z-score > 3.6, has p-value < 0.0002.

(a) List the hypothesis statement.

(1 mark)

(b) Indicate the appropriate test, its critical values and decision rules.

(3 marks)

(c) Implement suitable hypothesis testing on the researcher's claim and determine the conclusion.

(7 marks)

Q7 An engineer wishes to test the manufacturer's claim that the breaking strength of their steel wire has a variance of 0.6 N. The engineer believes that it is not the case. Assume that the breaking strength is normally distributed, the engineer sample 18 of the steel wires and observed a larger variability with variance of 1.0 N instead. Table of χ^2 shows: $\chi^2_{0.05,18} = 28.869$; $\chi^2_{0.95,18} = 9.390$; $\chi^2_{0.05,17} = 27.587$; $\chi^2_{0.025,17} = 30.191$; $\chi^2_{0.95,17} = 8.672$; $\chi^2_{0.975,17} = 7.564$.

(a) List the hypothesis statement.

(1 mark)

(b) Indicate the appropriate type of test, its critical value/score and decision rules.

(3 marks)

(c) Using χ^2 hypothesis testing at $\alpha = 0.05$, examine whether there is enough evidence to reject the manufacturer's claim.

(6 marks)

Q8 A company manufactures sensitive electronic devices to be used in a very wide temperature range. The company knows that increased temperature shortens the life time of the device, and a study is therefore performed in which the life time is determined as a function of temperature. The data for the sample is shown in **Table Q8.**

Table Q8 Results of Experiment

Temperature (°C)	20	30	40	50	60	70
Life time (Hours)	365	285	220	176	117	69

(a) Identify the dependent variable and independent variable.

(2 marks)

(b) Determine the regression equation for the data.

(12 marks)

(c) Estimate the life time if operational temperature is 80°C.

(2 marks)

- END OF QUESTIONS -

FINAL EXAMINATION

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$X \sim N(\mu, \sigma^2)$

$Z \sim N(0, 1)$

$\bar{X} \sim N(\mu, \frac{\sigma^2}{n})$ $Z = \frac{\bar{X} - \mu}{\sigma/\sqrt{n}} \sim N(0, 1)$

$Z = \frac{X - \mu}{\sigma}$

$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2})$ $Z = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1)$

$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t_{\alpha, n-1}$ $F = \frac{S_1^2}{S_2^2} \sim f_{\alpha, n_1-1, n_2-1}$

$\chi^2 = \frac{(n-1)S^2}{\sigma^2} \sim \chi^2_{\alpha, n-1}$ $s^2 = \frac{1}{n-1}\left(\sum x^2 - \frac{(\sum x)^2}{n}\right)$

$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{S_P\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} \sim t_{\alpha,\ n_1+n_2-2}$ $S_P^2 = \frac{(n_1-1)S_1^2 + (n_2-1)S_2^2}{n_1 + n_2 - 2}$

$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}} \sim t_{\alpha, \nu}$ $\nu = \frac{\left(S_1^2/n_1 + S_2^2/n_2\right)^2}{\frac{\left(S_1^2/n_1\right)^2}{(n_1-1)} + \frac{\left(S_2^2/n_2\right)^2}{(n_2-1)}}$ or $\nu = 2(n\text{-}1)$

$\hat{y} = \hat{\beta}_0 + \hat{\beta}_1 x$ $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

$\hat{\beta}_1 = \frac{S_{xy}}{S_{xx}}$

$\text{SSE} = S_{yy} - \hat{\beta}_1 S_{xy}$ $\text{MSE} = \frac{\text{SSE}}{n-2}$

$S_{xx} = \sum_{i=1}^{n} x_i^2 - \frac{\left(\sum_{i=1}^{n} x_i\right)^2}{n}$ $S_{yy} = \sum_{i=1}^{n} y_i^2 - \frac{\left(\sum_{i=1}^{n} y_i\right)^2}{n}$ $S_{xy} = \sum_{i=1}^{n} x_i y_i - \frac{\sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n}$