UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	TECHNICAL MATHEMATICS II
COURSE CODE	:	DAS 11103
PROGRAMME CODE	:	DAK
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS.
		2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**.
		3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA **CLOSED BOOK**.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) Sketch the graph of the following functions. hence determine the domain and range:

$$f(x)=\begin{cases} x^2-x+2 & ;x\le 1 \\ x^3+3 & ;1<x\le 7. \\ 4 & ;x>7 \end{cases}$$

(7 marks)

(b) Given the function, $f(x)=10^{2x}$.

(i) Determine the inverse function, $f^{-1}(x)$.

(3 marks)

(ii) Hence. sketch the graph of $f(x)=10^{2x}$ and its inverse.

(3 marks)

(c) If $f(x)=6x+2$, $g(x)=3^{-x}$ and $h(x)=2-x^2$, calculate:

(i) $(f\circ g)(x)$.

(2 marks)

(ii) $(f\circ g\circ h)(1)$.

(5 marks)

Q2 (a) Evaluate the following limit if it exists:

(i) $\lim\limits_{x\to 4}\dfrac{\frac{8}{x}-2}{x-4}$.

(2 marks)

(ii) $\lim\limits_{x\to 0}\dfrac{e^{1.5x}+1}{e^{1.5x}}$.

(2 marks)

(iii) $\lim\limits_{x\to -2}\dfrac{\sqrt{x^2+12}-4}{x+2}+\lim\limits_{x\to\infty}\dfrac{\sqrt{5}}{x}$.

(5 marks)

(b) Given that a function, $f(h)$ is written as:

$$f(h)=\begin{cases}\dfrac{h^2-9}{h-3} & ;h<3\\ -(h^2)+k & ;3\le h<8\\ -25 & ;h=8\\ 11-\dfrac{60}{8}h & ;h>8\end{cases}.$$

(i) Find the value of k if $\lim_{h\to 3} f(h)$ exist.

(5 marks)

(ii) Determine whether the function is continuous at $h=8$.

(6 marks)

Q3 (a) Differentiate the following function using implicit differentiation method:

$$x^2+3xy+y^3-8=0.$$

(4 marks)

(b) Evaluate the differential if $x=2t^3$ and $y=t^3+t^2$ using parametric differentiation.

(5 marks)

(c) Given a function as follows:

$$y=x^3+\frac{3}{2}x^2-6x+2.$$

(i) Determine the local extrema and fill out the **Table Q3(c)(i)**.

(8 marks)

(ii) Hence, sketch the graph.

(3 marks)

Q4 (a) Solve the following integrals:

(i) $\int_{3}^{4}\left(m^{2}+\sqrt[5]{m}-\frac{2}{m^{5}}\right)dm.$

(5 marks)

(ii) $\int\frac{\sin(\ln x)}{x}dx.$

(4 marks)

(iii) $\int\frac{x^{2}\cos(2x)}{x}dx.$

(4 marks)

(b) Evaluate the following by Simpsons Rule with $h=0.25$:

$$\int_{0}^{1}\sqrt{x^{2}+1}\,dx.$$

(7 marks)

Q5 (a) Given $y=x^{2}-2x+14$ and $y=-x^{2}+10x+4$.

(i) In the same Cartesian coordinate, sketch the graph of the functions above.

(6 marks)

(ii) Hence, find the area of the region bounded by the two curves.

(7 marks)

(b) For the region of $0.5\leq x\leq 3.5$, evaluate the length of arc of the following curve:

$$y=\frac{(x^{2}+2)^{\frac{3}{2}}}{3}.$$

(7 marks)

– END OF QUESTIONS –

FINAL EXAMINATION

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PROGRAMME CODE : DAK
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Table Q3(c)(i)

Critical and inflection points							
Test value							
f' behaviour							
f'' behaviour							

FINAL EXAMINATION

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LIST OF FORMULA

Table 1: Differentiation

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin u]=\cos u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos u]=-\sin u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[cu]=c\frac{du}{dx}$	$\frac{d}{dx}[\ln\|u\|]=\frac{1}{u}\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}\left[e^u\right]=e^u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[a^u]=a^u\ln a\left(\frac{du}{dx}\right)$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\tan u]=\sec^2 u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[\log_a u]=\frac{1}{u}\log_b e\left(\frac{du}{dx}\right)$	$\frac{d}{dx}[\sec u]=\sec u\tan u\left(\frac{du}{dx}\right)$
Chain Rule: $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$	Parametric Differentiation: $\frac{dy}{dx}=\left(\frac{dy}{dt}\Big/\frac{dx}{dt}\right)=\frac{dy}{dt}.\frac{dt}{dx}$

FINAL EXAMINATION

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Table 2: Integration

$\int a\,dx = ax + C$	$\int \sin nx\,dx = -\frac{1}{n}(\cos nx) + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, (n \neq -1)$	$\int \cos nx\,dx = \frac{1}{n}(\sin nx) + C$
$\int \frac{1}{nx+b}\,dx = \frac{1}{n}\ln\lvert nx+b\rvert + C$	$\int \tan x\,dx = \ln\lvert \sec x \rvert + C$
$\int \frac{1}{b-nx}\,dx = -\frac{1}{n}\ln\lvert b-nx\rvert + C$	$\int \sec^2 x\,dx = \tan x + C$
$\int e^{nx}\,dx = \frac{1}{n}\left(e^{nx}\right) + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int e^{nx+b}\,dx = \frac{1}{n}e^{nx+b} + C$	$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + C$
Integration part by part: $\int u dv = uv - \int v du$	
Improper Integral: $\int_a^{\infty} f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$	
Identity: $1 + \tan^2 x = \sec^2 x$	

Area of Region

$$A = \int_a^b \left[f(x) - g(x)\right]dx \qquad \text{or} \qquad A = \int_c^d \left[w(y) - v(y)\right]dy$$

Volume Cylindrical Shells

$$V = \int_a^b 2\pi x\, f(x)\, dx \qquad \text{or} \qquad V = \int_c^d 2\pi y\, f(y)\, dy$$

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Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{or} \quad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Partial Fraction

$$\frac{a}{x^2 - 1} = \frac{A}{x+1} + \frac{B}{x-1}$$

Simpson's Rule

$$\int_a^b f(x)\, dx \approx \frac{h}{3}\left[\left(f(a) + f(b)\right) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f(a+ih) + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f(a+ih) \right] ; \quad n = \frac{b-a}{h}$$

Trapezoidal Rule

$$\int_a^b f(x)\, dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(a+ih) \right] ; \quad n = \frac{b-a}{h}$$