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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	DAU 34403
PROGRAMME CODE	:	DAU
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 (a) Solve the following equation using separable equation: $2x\frac{dy}{dx}-1=3y+5\frac{dy}{dx}$.

(7 marks)

(b) Solve the following equation using linear equation: $x^3\frac{dy}{dx}+x^2y=1$.

(6 marks)

(c) Show that the following equation is an exact equation and hence solve the equation: $(2xy^2-9x^2)dx+(3y+2x^2y+1)dy=0$.

(7 marks)

Q2 During the semester break, you work at the Pagoh factory. You need to remove a metal with its core temperature of 1200 °F from a furnace and placed the metal on a table in a room that had a constant temperature of 73°F. One and half hour after it is removed the core temperature is 1000°F, when you check the temperature of the metal. The temperature of the metal must be below 500 °F before you can transfer it to the next section. You removed the metal at 8.00 am and your lunch start at 1 pm.

(a) Find the rate of change of the temperature dT/dt in term of T and T_s, given the temperature of the metal $T(t)$ and the ambient temperature T_s

(3 marks)

(b) Show that $T-T_s=Ae^{-kt}$.

(4 marks)

(c) Using the observed initial temperatures of the metal, $T(0)=1200$, find the constant A. Hence find $T(t)$.

(4 marks)

(d) Using the observed temperatures of the metal, given $T(1.5)=1000$, find the constant k.

(4 marks)

(e) If you removed the metal at 8.00 am, determine whether the metal will be transferred to the next section before or after your lunch.

(5 marks)

Q3 (a) Given $y'' - 9y = 0$.

(i) Compute the general solution of the homogeneous differential equations. (3 marks)

(ii) Compute the particular solution of the nonhomogeneous differential equations $y'' - 9y = 27x^2$. (3 marks)

(iii) Compute the particular solution of the nonhomogeneous differential equations $y'' - 9y = 5 \cos x$. (3 marks)

(iv) Thus compute the general solution of the homogeneous differential equations $y'' - 9y = 27x^2 + 5 \cos x$. (1 mark)

(b) Given a nonhomogeneous second order differential equation as below:
$y'' - 2y' + y = e^{-2x}$.

(i) Find the homogenous solution, y_h. (2 marks)

(ii) From y_h in **Q3(b) (i)**, find y_1, y_1', y_2 and y_2'. (2 marks)

(iii) Calculate the Wronskian function, W. (2 marks)

(iv) Find u_1 and u_2. (3 marks)

(v) Thus write the general solution of the equation. (1 mark)

Q4 (a) (i) Find the Laplace Transform for $3e^{-t} - \sinh 4t + 2t^5 + 6$. (4 marks)

(ii) By using the first shift property, find $L\{e^t \sin 5t\}$. (3 marks)

(iii) By using Multiply with t^n Property, find $L\{te^{2t}\}$. (3 marks)

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(b) Find the inverse of the following transforms

(i) $L^{-1}\left\{\dfrac{6}{s^2-4}\right\}$.

(2 marks)

(ii) $L^{-1}\left\{\dfrac{s+2}{s^2+4}\right\}$.

(3 marks)

(iii) $L^{-1}\left\{\dfrac{s+2}{(s-2)^2+9}\right\}$.

(5 marks)

Q5 (a) Solve the initial value problem $y''-y'=te^{2t}$ with $y(0)=1$.

(10 marks)

(b) Solve the boundary value problem $y''+y'=\cos 2t$ with $y(0)=1, y'(\pi/2)=-1$.

(10 marks)

- END OF QUESTIONS -

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Formula

Differentiation/Integration		
$\frac{d}{dx}(\sin ax) = a\cos ax$	$\frac{d}{dx}(e^{ax}) = ae^{ax}$	$\int e^{ax}dx = \frac{e^{ax}}{a} + C$
$\frac{d}{dx}(\cos ax) = -a\sin ax$	$\int \sin axdx = -\frac{\cos ax}{a} + C$	$\int (uv)dx = uv - \int vdu$
$\frac{d}{dx}(uv) = vu' + uv'$	$\int \cos axdx = \frac{\sin ax}{a} + C$	

First Order Differential Equations

Integration by Parts : $\int u\ dv = uv - \int v\ du$

Exact Equation: $M(x,y)dx + N(x,y)dy = 0$ and $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$.

Linear Equations: $\frac{dy}{dx} + p(x)y = q(x)$ and integrating factor $\rho(x) = e^{\int p(x)dx}$

Second Order Differential Equations

Differential equation $ay'' + by' + cy = 0$;

Characteristic equation : $am^2 + bm + c = 0$, $m_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Case	Roots of the Characteristic Equation	General Solution
1	real and distinct : $m_1 \neq m_2$	$y_h(x) = Ae^{m_1 x} + Be^{m_2 x}$
2	real and equal : $m_1 = m_2 = m$	$y_h(x) = (A + Bx)e^{mx}$
3	imaginary : $m = \alpha \pm i\beta$	$y_h(x) = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Method of Undetermined Coefficients

Case	Format of Non-homogeneous Term $r(x)$	Trial Function for Particular Solution $y_p(x)$
1	$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \ldots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \ldots + B_1 x + B_0)$
2	$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
3	$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

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Variation of Parameters Method:

$ay''+by'+cy=f(x)$

Homogeneous solution, $y_h(x)=Ay_1+By_2$;

Wronskian function, $W=\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}=y_1y_2'-y_2y_1'$,

$u_1=-\int \frac{y_2 f(x)}{aW}dx+A$; $u_2=\int \frac{y_1 f(x)}{aW}dx+B$;

General solution, $y(x)=u_1y_1+u_2y_2$

Table of Laplace Transforms

$L\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
e^{at}	$\frac{1}{s-a}$
$t^n,\ n=1,2,3,...$	$\frac{n!}{s^{n+1}}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sin at$	$\frac{a}{s^2+a^2}$

$L\{f(t)\}=\int_0^\infty f(t)e^{-st}dt=F(s)$	
$f(t)$	$F(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$e^{at}f(t)$	$F(s-a)$
$t^n f(t),\ n=1,2,3,...$	$(-1)^n\frac{d^nF}{ds^n}$

Application of Laplace Transforms

If $L\{y(t)\}=Y(s)$ then

$L\{y'(t)\}=sY(s)-y(0)$ and $L\{y''(t)\}=s^2Y(s)-sy(0)-y'(0)$