UTHM
Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	DIFFERENTIAL EQUATIONS
COURSE CODE	:	BDA 24303
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ONLY **THREE** (3) QUESTIONS IN **PART A** AND **ALL** QUESTIONS IN **PART B**
		2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**
		3. STUDENTS ARE **PROHIBITED** FROM CONSULTING THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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PART A: ANSWER ONLY **THREE (3)** FROM **FOUR (4)** QUESTIONS

Q1 (a) Solve the following initial value problem:

$$(x^2 + 2)\frac{dy}{dx} + (2xy + 9x^2) = 0$$
$$y(0) = 3$$

(9 marks)

(b) Show that the following equation is homogeneous and then solve for the general solution:

$$2yt\frac{dy}{dt} - 3y^2 - t^2 = 0$$

(11 marks)

Q2 (a) Find the particular solution for the following equation satisfying the given initial conditions:

$$y'' + 4y' + 5y = 0$$
$$y(0) = 1, y'(0) = 1$$

(8 marks)

(b) Solve the following equation using the method of Variation of Parameters:

$$y'' + y' - 6y = 2e^x$$

(12 marks)

Q3 (a) For the function shown in **Figure Q3(a)**, express in terms of unit step function and find the Laplace transform.

(12 marks)

(b) Find the inverse of the following Laplace function:

$$F(s) = \frac{e^{-s}}{(s^2 + 1)(s^2 + 4)}$$

(8 marks)

Q4 Solve the following initial value differential equation using Laplace transform:

$$2y'' + 8y' + 6y = 4e^{-2t} \qquad y(0) = 1, y'(0) = 2$$

(20 marks)

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PART B: ANSWER **ALL** QUESTIONS

Q5 Let $f(x)$ be a function of period 2π such that:

$$f(x) = \begin{cases} \pi - x, & 0 < x < \pi \\ 0, & \pi < x < 2\pi \end{cases}$$

(a) Sketch the graph of $f(x)$ in the interval $-2\pi < x < 2\pi$

(2 marks)

(b) Prove that the Fourier series for $f(x)$ in the interval $0 < x < 2\pi$ is:

$$f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right] + \left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$$

(12 marks)

(c) By giving an appropriate value x, demonstrate that
$$\frac{\pi^2}{8} = 1 + \frac{1}{9} - \frac{1}{25} + \frac{1}{49} - \cdots$$

(6 marks)

Q6 A 3 cm length silver bar with constant cross section area of 1 cm^2, density of 10 g/cm^3, thermal conductivity of 3 cal/cm.sec.°C and specific heat of 0.15 cal.g.°C, is perfectly insulated laterally with ends kept at temperature 0 °C and initial uniform temperature $f(x) = 25$ °C. The heat equation is:

$$\frac{\partial^2 u}{\partial x^{2x}} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

where u is temperature and t is time.

(a) Show that $c^2 = 2$

(2 marks)

(b) By using the method of separation of variable, and applying the boundary condition, prove that:

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3} e^{-\frac{2n^2\pi^2 t}{9}}$$

where b_n is an arbitrary constant.

(12 marks)

(c) By applying the initial condition, find the value of b_n

(6 marks)

- END OF QUESTIONS –

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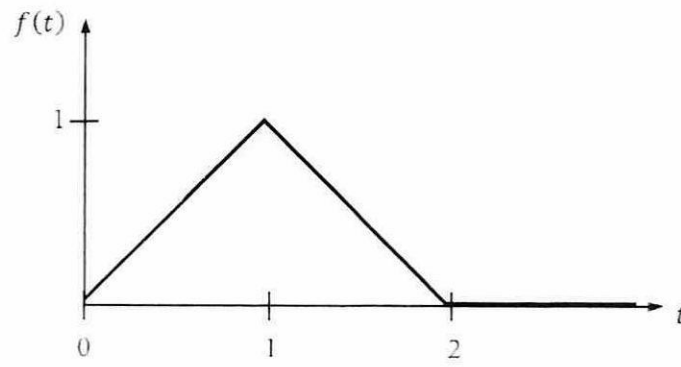|	
SEMESTER/SESSION : SEM II 2022/2023	PROGRAMME CODE : BDD
COURSE NAME : DIFFERENTIAL EQUATIONS	COURSE CODE : BDA 24303

Figure Q3(a)

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SEMESTER/SESSION : SEM II 2022/2023	PROGRAMME CODE : BDD
COURSE NAME : DIFFERENTIAL EQUATIONS	COURSE CODE : BDA 24303

FORMULAS

First Order Differential Equations

Type of ODEs	General solution
Linear ODEs: $y' + P(x)y = Q(x)$	$y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx} Q(x)dx + C\right\}$
Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$	$F(x,y) = \int f(x,y)dx$ $F(x,y) - \int \left\{\frac{\partial F}{\partial y} - g(x,y)\right\}dy = C$
Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$ $\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$ Integrating factor; $i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$ $i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$	$\int iM(x,y)dx - \int\left\{\frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y)\right\}dy = C$

Characteristic Equation and General Solution for Second Order Differential Equations

Types of Roots	General Solution
Real and Distinct Roots: m_1 and m_2	$y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$
Real and Repeated Roots: $m_1 = m_2 = m$	$y = c_1 e^{mx} + c_2 x e^{mx}$
Complex Conjugate Roots: $m = \alpha \pm i\beta$	$y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$

Method of Undetermined Coefficients

$g(x)$	y_p
Polynomial: $P_n(x) = a_n x^n + ... + a_1 x + a_0$	$x^r(A_n x^n + ... + A_1 x + A_0)$
Exponential: e^{ax}	$x^r(Ae^{ax})$
Sine or Cosine: $\cos \beta x$ or $\sin \beta x$	$x^r(A \cos \beta x + B \sin \beta x)$

Note: r is 0, 1, 2 ... in such a way that no terms in $y_p(x)$ is similar to those in $y_c(x)$.

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FINAL EXAMINATION

SEMESTER/SESSION   : SEM II 2022/2023   PROGRAMME CODE   : BDD
COURSE NAME   : DIFFERENTIAL EQUATIONS   COURSE CODE   : BDA 24303

Method of Variation of Parameters

The particular solution for $y'' + by' + cy = g(x)$ with b and c as constants is:
$$y_p(x) = u_1 y_1 + u_2 y_2$$

where,

$$u_1 = -\int \frac{y_2 g(x)}{W} dx \quad \& \quad u_2 = \int \frac{y_1 g(x)}{W} dx$$

where,

$$W = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix}$$

Laplace Transform

$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st} dt = F(s)$	
$f(t)$	$F(s)$
a	$\dfrac{a}{s}$
$t^n, n = 1, 2, 3, \ldots$	$\dfrac{n!}{s^{n+1}}$
e^{at}	$\dfrac{1}{s-a}$
$\sin at$	$\dfrac{a}{s^2 + a^2}$
$\cos at$	$\dfrac{s}{s^2 + a^2}$
$\sinh at$	$\dfrac{a}{s^2 - a^2}$
$\cosh at$	$\dfrac{s}{s^2 - a^2}$
$e^{at} f(t)$	$F(s-a)$
$t^n f(t), n = 1, 2, 3, \ldots$	$(-1)^n \dfrac{d^n F(s)}{ds^n}$

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FINAL EXAMINATION

Laplace Transform (continued)

$H(t-a)$	$\dfrac{e^{-as}}{s}$
$f(t-a)H(t-a)$	$e^{-as}F(s)$
$f(t)\delta(t-a)$	$e^{-as}f(a)$
$y(t)$	$Y(s)$
$\dot{y}(t)$	$sY(s)-y(0)$
$\ddot{y}(t)$	$s^2Y(s)-sy(0)-y'(0)$

Partial Fraction Expansion

Factor in denominator	Term in partial fraction decomposition
$ax+b$	$\dfrac{A}{ax+b}$
$(ax+b)^k$	$\dfrac{A_1}{ax+b}+\dfrac{A_2}{(ax+b)^2}+\cdots+\dfrac{A_k}{(ax+b)^k},\ k=1,2,3,\ldots$
ax^2+bx+c	$\dfrac{Ax+B}{ax^2+bx+c}$
$(ax^2+bx+c)^k$	$\dfrac{A_1x+B_1}{ax^2+bx+c}+\dfrac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\dfrac{A_kx+B_k}{(ax^2+bx+c)^k},\ k=1,2,3,\ldots$

Fourier Series

Fourier series expansion of periodic function with period 2π

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\cos nx\,dx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi}f(x)\sin nx\,dx$$

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

Half Range Series

$$a_0=\frac{2}{L}\int_{0}^{L}f(x)dx$$

$$a_n=\frac{2}{L}\int_{0}^{L}f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n=\frac{2}{L}\int_{0}^{L}f(x)\sin\frac{n\pi x}{L}\,dx$$

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$$

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FINAL EXAMINATION

SEMESTER/SESSION	: SEM II 2022/2023	PROGRAMME CODE	: BDD
COURSE NAME	: DIFFERENTIAL EQUATIONS	COURSE CODE	: BDA 24303

Trigonometric Identities

TANGENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

RECIPROCAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

PERIODIC IDENTITIES

$$\sin(\theta + 2\pi n) = \sin\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta$$

$$\tan(\theta + \pi n) = \tan\theta$$

$$\csc(\theta + 2\pi n) = \csc\theta$$

$$\sec(\theta + 2\pi n) = \sec\theta$$

$$\cot(\theta + \pi n) = \cot\theta$$

EVEN/ODD IDENTITIES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

HALF ANGLE IDENTITIES

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

PRODUCT TO SUM IDENTITIES

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

SUM TO PRODUCT IDENTITIES

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha-\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

LAW OF TANGENTS

$$\frac{a-b}{a-b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$

$$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$$

$$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$$

SUM/DIFFERENCES IDENTITIES

$$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha\pm\beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

MOLLWEIDE'S FORMULA

$$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha-\beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$$

COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

LAW OF SINES

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$

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