UTHM
Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2022/2023

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|---|---|---|
| COURSE NAME | : | CALCULUS FOR ENGINEER |
| COURSE CODE | : | BDA 14403 |
| PROGRAMME CODE | : | BDD |
| EXAMINATION DATE | : | JULY/AUGUST 2023 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | 1. ANSWER **FIVE (5)** QUESTIONS **ONLY**.<br>2. THIS FINAL EXAMINATION IS CONDUCTED VIA CLOSED BOOK.<br>3. STUDENTS ARE PROHIBITED TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK. |

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

**Q1** (a) Determine the domain and range of the given functions. Then sketch the domain.

$$f(x,y)=\frac{1}{\sqrt{36-y^2-x^2}}$$

(6 marks)

(b) Plot a three-dimensional (3D) graph of the following function:

$$z=x^2+y^2+3$$

(3 marks)

(c) Show that the following limit does not exists by considering the limit along two different smooth curves or lines:

$$\lim_{(x,y)\to(0,0)}\frac{x}{x+y}$$

(7 marks)

(d) Determine whether the following function continuous at point (0, 0). Justify your answer.

$$f(x,y)=\frac{\cos xy}{2+\sin y}$$

(4 marks)

**Q2** (a) If $z=x^3\cos\frac{y}{x}$, find $z_x$ and $z_y$.

(4 marks)

(b) Show that $f(x,y)=\sqrt{x^4+y^4}$ satisfies the equation $x\frac{df}{dx}+y\frac{df}{dy}=2f$.

(4 marks)

(c) If $z=3x^2y+xy^4$, where $x=\sin 2t$ and $y=\cos t$, find $\frac{dz}{dt}$ and $\frac{d^2z}{dt^2}$ when $t=0$.

(6 marks)

(d) Gas escapes from a spherical balloon at $2\,\text{m}^3\,\text{min}^{-1}$. How fast is the surface area shrinking when the radius equals 12m? The surface area of a sphere of radius $r$ is $4\pi r^2$.

(6 marks)

**Q3** (a) Solve $\int_0^2 \frac{x}{(x+1)^3} dx$

(5 marks)

(b) By changing to polar coordinate, evaluate $\iint_R x^2 y dA$, where $R$ is the upper half of the annulus bounded in between $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$

(7 marks)

(c) By using double integrals, find the surface area of the portion of hemisphere $z = \sqrt{16 - x^2 - y^2}$ that lies above the region R by the disc $x^2 + y^2 \leq 4$.

(8 marks)

**Q4** (a) Calculate the volume of the solid enclosed by a paraboloid $z = x^2 + y^2$ and plane $z = 16$ in first octant.

(6 marks)

(b) Evaluate

$$\int_0^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} x^2 + y^2 + z^2 \, dz \, dy \, dx$$

by changing to spherical coordinate.

(6 marks)

(c) Find the moment of mass about *xy*-plane for solid $G$ bounded below by *xy*-plane, above by $z = \sqrt{8 - x^2 - y^2}$ and sided by cylinder $x^2 + y^2 = 9$, with the density function is given by $\delta(x, y, z) = 2$.

(8 marks)

**Q5** (a) Solve the integral $\int_C (2x + y)\, dx + (5x - y)\, dy$ along the curve $x = y^3$ from $(0,0)$ to $(1, 1)$.

(6 marks)

(b) Given the force field $\boldsymbol{F}(x, y) = \left(6x^2 - 2xy^2 + \frac{y}{2\sqrt{x}}\right)\mathbf{i} - \left(2x^2 y - 4 - \sqrt{x}\right)\mathbf{j}$. Show that $\boldsymbol{F}(x, y)$ is conservative and then find the potential function which satisfied $\nabla\emptyset = \boldsymbol{F}$.

(6 marks)

(c) Use Green's Theorem to evaluate $\int_C (xy^2 + x^2)\, x + (4x - 1)\, dy$, where $C$ is a the closed triangular path from origin to (0,3), (-3, 0) and back to origin in that order.

(8 marks)

**Q6** (a) If $F = x^2yz\,\underline{i} + xy^2z\,\underline{j} + xyz^2\,\underline{k}$, find Curl $F$ at the point $(1,2,3)$.

(6 marks)

(b) Find the surface area of $x + 3y + z = 6$ in the first octant.

(6 marks)

(c) Evaluate

$$\iint_{\sigma} F.n\,dS$$

by using Gauss's Theorem where $F(x, y, x) = xy^2\,\underline{i} + yz^2\,\underline{j} + x^2z\,\underline{k}$ and $\sigma$ is the surface bounded above by sphere $x^2 + y^2 + z^2 = 4$ and below by cone $z = \sqrt{x^2 + y^2}$.

(8 marks)

**-END OF QUESTION-**

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**FORMULA**

**Total Differential**
For function $z = f(x, y)$, the total differential of $z$, $dz$ is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

**Relative Change**
For function $z = f(x, y)$, the relative change in $z$ is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

**Rate of Change**
For function with single variable, $y(x)$, the rate of change is given by

$$\frac{dy}{dx} = \frac{dy}{dx}\frac{dx}{dt}$$

For function with two variables, $z = f(x, y)$, the rate of change is given by

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

**Increment of Approximation**
For a function of two variables, , $z = f(x, y)$, when , $(x, y)$ moves from the initial point , $(x_0, y_0)$ to $(x_1, y_1)$, the following can be calculated by using total differential and partial derivative;

***Approximate change*** in z;

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

***Exact change*** in z;

$$\Delta z = f(x_1, y_1) - f(x_0, y_0)$$

***Approximate value*** of z;

$$z = f(x_0, y_0) + dz$$

***Exact value*** of z;

$$z = f(x_1, y_1)$$

**Implicit Differentiation**
Suppose that $z$ is given implicitly as a function $z = f(x, y)$ by an equation of the form $F(x, y, z) = 0$, where $F(x, y, f(x, y)) = 0$ for all $(x, y)$ in the domain of $f$, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

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**Extreme of Function with Two Variables**

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
$f(x,y)$ has a local maximum value at $(a,b)$

b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
$f(x,y)$ has a local minimum value at $(a,b)$

c. If $D < 0$
$f(x,y)$ has a saddle point at $(a,b)$

d. If $D = 0$
The test is inconclusive

**Surface Area**

$$\text{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

**Polar Coordinates:**

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\,r\,dr\,d\theta$$

**Cylindrical Coordinates:**

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$$

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**Spherical Coordinates:**

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

**In 2-D: Lamina**

Given that $\delta(x,y)$ is a density of lamina

**Mass**, $m = \iint_R \delta(x,y)\,dA$, where

**Moment of Mass**

a. About $x$-axis, $M_x = \iint_R y\delta(x,y)\,dA$

b. About $y$-axis, $M_y = \iint_R x\delta(x,y)\,dA$

**Centre of Mass**

Non-Homogeneous Lamina:

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

**Centroid**

Homogeneous Lamina:

$$\bar{x} = \frac{1}{Area\ of\ \ R}\iint_R x\,dA \text{ and } \bar{y} = \frac{1}{Area\ of\ \ R}\iint_R y\,dA$$

**Moment Inertia:**

a. $I_y = \iint_R x^2\delta(x,y)\,dA$

b. $I_x = \iint_R y^2\delta(x,y)\,dA$

c. $I_o = I_z = \iint_R (x^2+y^2)\delta(x,y)\,dA = I_x + I_y$

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**In 3-D: Solid**
Given that $\delta(x, y, z)$ is a density of solid

**Mass**, $m = \iiint_G \delta(x, y, z)dV$

If $\delta(x, y, z) = c$, where $c$ is a constant, $m = \iiint_G dA$ is volume.

**Moment of Mass**

a. About $yz$-plane, $M_{yz} = \iiint_G x\delta(x, y, z)dV$

b. About $xz$-plane, $M_{xz} = \iiint_G y\delta(x, y, z)dV$

c. About $xy$-plane, $M_{xy} = \iiint_G z\delta(x, y, z)dV$

**Centre of Gravity**

$$(\bar{x}, \bar{y}, \bar{z}) = \left( \frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m} \right)$$

**Moment Inertia**

a. About $x$-axis, $I_x = \iiint_G (y^2 + z^2)\delta(x, y, z)dV$

b. About $y$-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x, y, z)dV$

c. About $z$-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x, y, z)dV$

**Directional Derivative**

$D_u f(x, y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot \mathbf{u}$

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**Del Operator**

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

**Gradient** of $\phi = \nabla \phi$

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla . \mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let $C$ is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The **Binormal Vector,** $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

**Curvature**

$$\kappa = \frac{\|\mathbf{T}'(t)\|}{\|\mathbf{r}'(t)\|}$$

**Radius of Curvature**

$$\rho = \frac{1}{\kappa}$$

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**Green's Theorem**

$$\oint_C M dx + N dy = \iint_R \left( \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

**Gauss's Theorem**

$$\iint_S \mathbf{F} \cdot \mathbf{n} dS = \iiint_G \nabla \cdot \mathbf{F} dV$$

**Stoke's Theorem**

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n} dS$$

**Arc Length**

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a, b]$, hence, the **arc length**,

$$s = \int_a^b \| \mathbf{r}'(t) \| \, dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2} \, dt$$