UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL
COURSE CODE	:	BDU 21403
PROGRAMME CODE	:	BDC / BDM
EXAMINATION DATE	:	JULY/AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **FOUR (4)** QUESTIONS **ONLY.** 2. THIS FINAL EXAMINATION CONDUCTED VIA **CLOSED BOOK.** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

Q1 A tailless UAV is an unmanned aircraft that uses elevons to control its roll and pitch motion. The unmanned aircraft has poor damping in its roll dynamics. The roll control system for the UAV shown in **Figure Q1** is to be used to increase the system damping.

(a) Design the appropriate gain (K_a and K_r) so that the system meets the following performance specifications:

$$\xi = 0.8$$
$$T_s \leq 2 \text{ seconds}$$

(15 marks)

(b) Estimate the steady state error of your control design if the step input command reference, $R(s) = 10/s$ is applied to the control system.

(10 marks)

Q2 (a) Give two reasons for modeling systems in state space.

(2 marks)

(b) What factors influence the choice of state variables in any system?

(3 marks)

(c) Convert the transfer function given below to state-space representation:

$$\frac{Y(s)}{U(s)} = \frac{50}{s^3 + 15s^2 + 30s + 50}$$

Assume zero initial condition with output equation, $y(t) = x_1$.

(13 marks)

(d) Determine the eigenvalues of the system in question **Q2(c).**

(3 marks)

(e) In your opinion, can the transient response of the current dynamic system under consideration to be approximate as second order system? Give specific condition where a dynamic system with more than two poles can be approximated as a second order system.

(4 marks)

Q3 (a) Describe the physical features of the Short Period stability mode.

(3 marks)

(b) The Convair CV-880 was a 130-passenger, four-engine civil transport aircraft that first flew in 1960. The reduced-order state-space model corresponding to the short-period mode approximation for a general aviation aircraft is given as follows:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & Z_q \\ M_w & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_e$$

with the following stability derivatives:

$Z_w = -4.8210$ $M_q = -5.9983$
$Z_q = -3.7091$ $Z_{\delta_e} = -0.1456$
$M_w = -0.6492$ $M_{\delta_e} = -0.4408$

(i) Determine the characteristic equation of the short period mode.
(5 marks)

(ii) Determine the eigenvalues of the short period mode.
(2 marks)

(iii) Determine the damping ratio, natural frequency, period, time to half amplitude, and the number of cycles to half amplitude for the short period mode.
(5 marks)

(c) A helicopter based unmanned aerial vehicle (UAV), as shown in **Figure Q3(c)** is an unmanned helicopter that uses its cyclic pitch mechanism to change its pitch motion. The pitch angle to the longitudinal cyclic transfer function of the helicopter UAV can be modeled according to:

$$\frac{\theta(s)}{\delta_{lon}(s)} = \frac{19.5}{s(s+3.25)}$$

Design a pitch control system so that the UAV can exhibit pitch angle tracking performance with a desired damping ratio of $\xi = 1$, setting time, $t_s \leq 5$ s and no steady state error. Consider the sensor used in the control system design to be a perfect device.
(10 marks)

Q4 (a) Describe the functions of the following controllers:
i. Proportional (P) controller
ii. Proportional-Integral (PI) Controller
iii. Proportional-Integral-Derivative (PID) Controller
(3 marks)

(b) A business jet with a 16.4 m wingspan and a gross weight of 17,300 kg has the following transfer function for pitch angle to the elevator input:

$$G(s) = \frac{2(s+5)}{s^3 + 4s^2 + 5.3s + 6.9}$$

The pitch control system that will be designed for the business jet is shown in **Figure Q4(b)** with transfer functions for controller gain given as $K(s)$. If the controller's transfer function is set with **proportional gain only**, examine the closed-loop pole movement of the pitch control system if the controller gain varies from 0 to ∞. Determine the damped frequency, ω_d and gain, K, values at the imaginary axis crossing if such a situation exists. Discuss whether the approach of proportional gain design is suitable for pitch control system design.
(10 marks)

(c) Develop the automatic controllers (i.e., PI, PD, and PID control) for the dynamic system under consideration using the Ziegler and Nichols tuning method. Compare the steady-state error performances of the compensated systems (i.e., PI, PD, and PID control). Describe any problems with your design.

(12 marks)

Q5 (a) Provide a simple root locus sketch for the dynamic system given in **Figure Q5(a).**

(5 marks)

(b) The MP2000 is an unmanned aerial vehicle (UAV) with dimensions of 1.75 m in wingspan, 1.4 m in length, 0.5116 m^2 in wing area, 0.29 m in wing chord, and 3.84 kg in total mass. The UAV is equipped with a 1 HP glow engine and a miniature autopilot that can execute autonomous operations. To perform missions, the UAV must be able to maintain or maneuver to a commanded speed, altitude, and attitude angles. Consider the pitch angle control system for the UAV can be represented by the block diagram in **Figure Q5(b)**. The pitch angle to elevator transfer function for the UAV is given as follows:

$$G(s) = \frac{2(s+5)(s+14)}{s(s+9)(s^2+8s+25)}$$

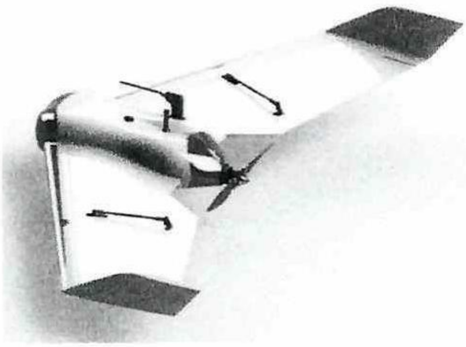
Suggest a value of gain, *K*, that results in a system with a damping ratio for the complex roots equal to 0.7. Provide a detailed root locus plot for the closed-loop system as *K* varies from 0 to ∞ with the necessary calculation to support your answer. Determine the settling time and overshoot for the system when the damping ratio of the complex roots is equal to 0.7.

(20 marks)

-END OF QUESTION-

FINAL EXAMINATION

SEMESTER/SESSION	: SEM II / 2022/2023	PROGRAMME CODE	: BDC/BDM
COURSE NAME	: AIRCRAFT STABILITY AND CONTROL	COURSE CODE	: BDU 21403



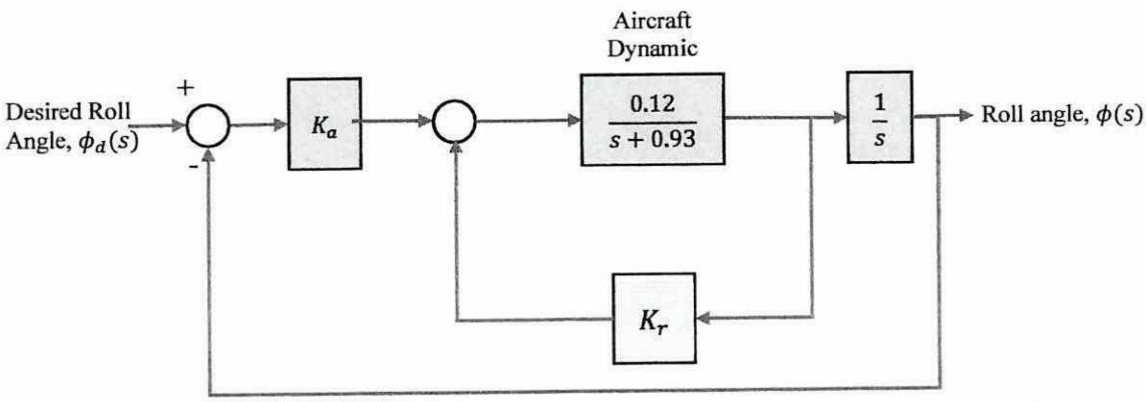


Figure Q1 Roll angle control system for a tailless unmanned aerial vehicle (UAV)

Figure Q3(c) Helicopter based unmanned aerial vehicle (UAV).

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM II / 2022/2023	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

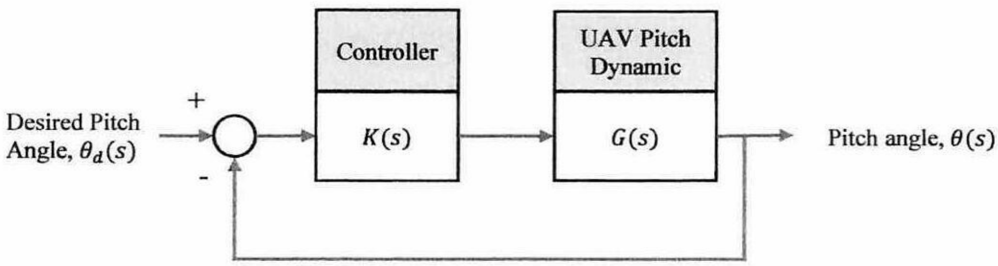


Figure Q4(b) Simplified block diagram for pitch angle control system.

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM II / 2022/2023	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

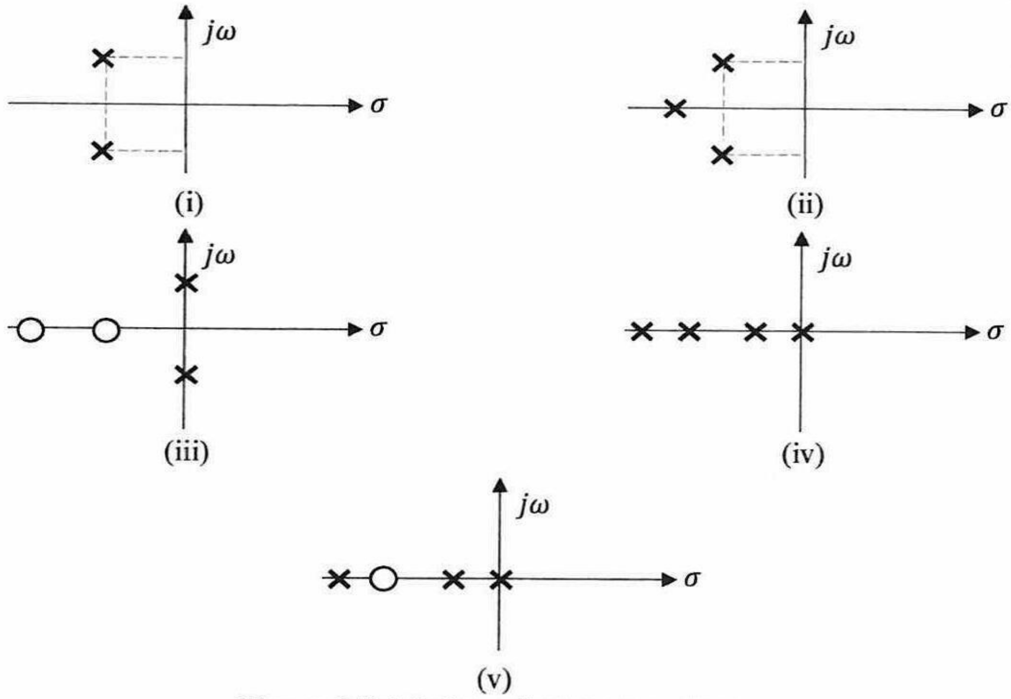


Figure Q5(a) Poles and zeros on s-plane.

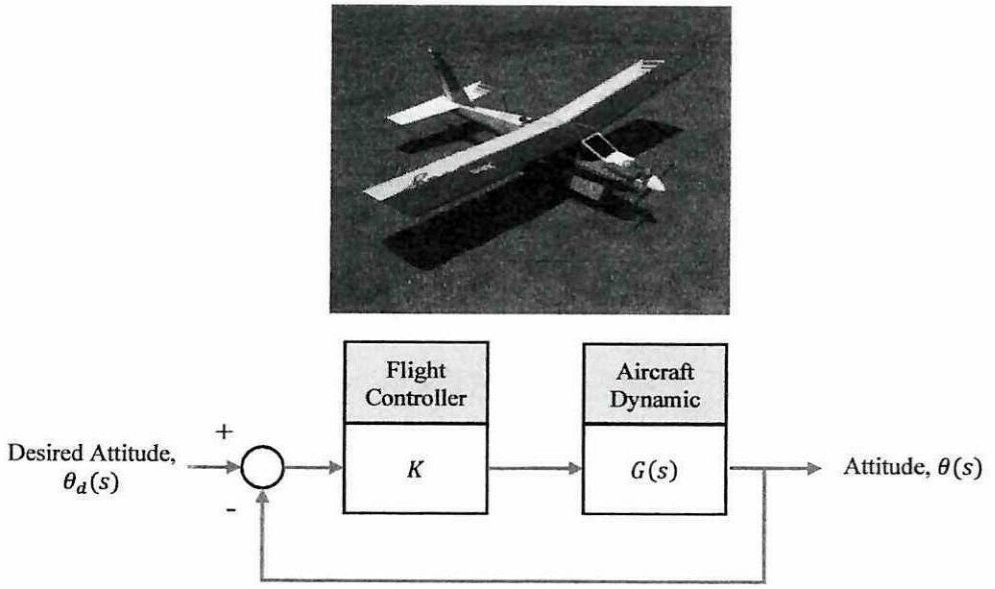


Figure Q5(b) The block diagram for the pitch control system for MP200 UAV.

FINAL EXAMINATION

SEMESTER/SESSION	: SEM II / 2022/2023	PROGRAMME CODE	: BDC/BDM
COURSE NAME	: AIRCRAFT STABILITY AND CONTROL	COURSE CODE	: BDU 21403

Key Equations

The relevant equations used in this examination are given as follows:

1. The determinant of a 3×3 matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad (1)$$

2. Partial fraction for $F(s)$ with real and distinct roots in the denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2}{(s+p_2)} + \cdots + \frac{K_m}{(s+p_m)} \quad (2)$$

3. Partial fraction for $F(s)$ with complex or imaginary roots in the denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2 + as + b)} + \cdots \quad (3)$$

4. General first-order transfer function:

$$G(s) = \frac{K}{s+a} \quad (4)$$

5. General second-order transfer function:

$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (5)$$

6. The closed-loop transfer function:

$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad (6)$$

where $G(s)$ is the transfer function of the open-loop system, and $H(s)$ is the transfer function in the feedback loop.

7. The final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (7)$$

8. Time response:

$$T_r = \frac{2.2}{a} \quad (8)$$

$$T_s = \frac{4}{a} \quad (9)$$

$$\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\% \quad (10)$$

$$\xi = \frac{-\ln\left(\%\frac{OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\%\frac{OS}{100}\right)\right)^2}} \quad (11)$$

FINAL EXAMINATION

SEMESTER/SESSION	: SEM II / 2022/2023	PROGRAMME CODE	: BDC/BDM
COURSE NAME	: AIRCRAFT STABILITY AND CONTROL	COURSE CODE	: BDU 21403

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega_d} \quad (12)$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\sigma} \quad (13)$$

$$P = \frac{2\pi}{\omega_d} \quad (14)$$

$$t_{1/2} = \frac{0.693}{|\sigma|} \quad (15)$$

$$N_{1/2} = 0.110\frac{|\omega_d|}{|\sigma|} \quad (16)$$

9. Estimation of parameter q (Paynter Numerical Method)

$$q = \max\left|A_{ij}\Delta t\right| \quad (17)$$

10. Estimation of the integer value of p (Paynter Numerical Method)

$$\frac{1}{p!}(nq)^p e^{nq} \leq 0.001 \quad (18)$$

11. Numerical solution of state equation:

$$\mathbf{x}_{k+1} = \boldsymbol{M}\mathbf{x}_k + \boldsymbol{N}\boldsymbol{u}_k$$

with matrix $\boldsymbol{M}$ and $\boldsymbol{N}$ are given by the following matrix expansion:

$$\boldsymbol{M} = e^{A\Delta t} = \boldsymbol{I} + A\Delta t + \frac{1}{2!}\boldsymbol{A}^2\Delta t^2 \ldots \quad (19)$$

$$\boldsymbol{N} = \Delta t\left(\boldsymbol{I} + \frac{1}{2!}A\Delta t + \frac{1}{3!}\boldsymbol{A}^2\Delta t^2 + \cdots\right)\boldsymbol{B}$$

12. Characteristic equation of the closed loop system:

$$1 + KG(s)H(s) = 0 \quad (20)$$

13. Asymptotes: angle and real-axis intercept:

$$\sigma = \frac{[\sum Real\ parts\ of\ the\ poles - \sum Real\ parts\ of\ the\ zeros]}{n-m} \quad (21)$$

$$\phi_a = \frac{180°[2q+1]}{n-m} \quad (22)$$

14. The solution to determine real axis break-in and breakaway points:

$$\frac{dK(s)}{ds} = 0 \quad (23)$$

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM II / 2022/2023	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU21403

15. An alternative solution to find real axis break-in and breakaway points:

$$\sum \frac{1}{s+z_i} = \sum \frac{1}{s+p_i} \quad (24)$$

16. The angle of departure of the root locus from a pole of $G(s)H(s)$:

$$\theta = 180° + \sum (angles\ to\ zeros) - \sum (angles\ to\ poles) \quad (25)$$

17. The angle of arrival at a zero:

$$\theta = 180° - \sum (angles\ to\ zeros) + \sum (angles\ to\ poles) \quad (26)$$

18. The steady-state error:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) \quad (27)$$

where the error signal is given as:

$$E(s) = \frac{1}{1+K(s)G(s)H(s)} \times U(s) \quad (28)$$

19. The characteristic equation for the standard form of the second-order differential equation:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

The roots of the characteristic equation are: (29)

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1-\xi^2} \cdot i$$

$$s_{1,2} = \sigma \pm \omega_d$$

20. The calculation of controller gains using the Ziegler-Nichols method:

Table 1 The Ziegler-Nichols tuning method. (30)

Control Type	K_p	K_I	K_D
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2\,K_p/T_u$	-
PD	$0.8K_u$	-	$K_PT_u/8$
Classic PID	$0.6K_u$	$2\,K_p/T_u$	$K_PT_u/8$
Pessen Integral Rule	$0.7K_u$	$2.5\,K_p/T_u$	$3K_PT_u/20$
Some Overshoot	$0.33K_u$	$2\,K_p/T_u$	$K_PT_u/3$
No Overshoot	$0.2K_u$	$2\,K_p/T_u$	$K_PT_u/3$

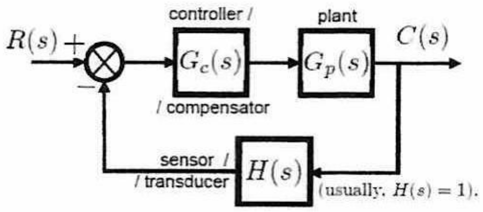
21. Conversion from the state-space model to transfer function model: (31)

$$G(s) = \boldsymbol{C}\frac{adj(s\boldsymbol{I}-\boldsymbol{A})}{\det(s\boldsymbol{I}-\boldsymbol{A})}\boldsymbol{B}$$

FINAL EXAMINATION

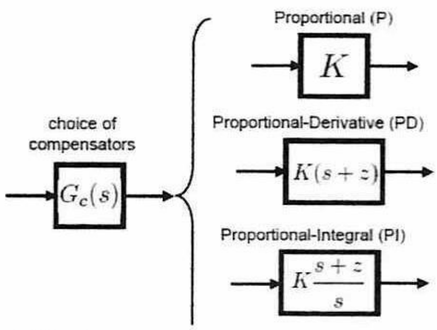
SEMESTER/SESSION	:	SEM II / 2022/2023	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU21403

22. Compensator design using the Root Locus:



generic system block diagram with controller/compensator and feedback

purpose: to improve the system's dynamics by proper choice of the controller TF and gain



Open–Loop TF: $KG_p(s)G_c(s)H(s)$ Closed–Loop TF: $\dfrac{KG_p(s)G_c(s)}{1+KG_p(s)G_c(s)H(s)}$

23. Root Locus sketching rules:

1. The root locus contours are symmetrical about the real axis.
2. The number of separate branches of the root locus plot is equal to the number of poles of the transfer function $G(s)H(s)$. Branches of the root locus originate at the poles of $G(s)H(s)$ for $k = 0$ and terminate at either the open-loop zeroes or at infinity for $k = \infty$. The number of branches that terminate at infinity is equal to the difference between the number of poles and zeroes of the transfer function $G(s)H(s)$, where n = number of poles and m = number of zeros.
3. Segments of the real axis that are part of the root locus can be found in the following manner: Points on the real axis that have an odd number of poles and zeroes to their right are part of the real axis portion of the root locus.
4. The root locus branches that approach the open-loop zeroes at infinity do so along straight-line asymptotes that intersect the real axis at the center of gravity of the finite poles and zeroes. Mathematically this can be expressed as

   $$\sigma = \left[\sum \text{Real parts of the poles} - \sum \text{Real parts of the zeroes}\right] \Big/ (n-m)$$

   where n is the number of poles and m is the number of finite zeroes.
5. The angle that the asymptotes make with the real axis is given by

   $$\phi_a = \frac{180^\circ[2q+1]}{n-m}$$

   for $q = 0, 1, 2, \ldots, (n-m-1)$
6. The angle of departure of the root locus from a pole of $G(s)H(s)$ can be found by the following expression:

   $$\phi_p = \pm 180^\circ(2q+1) + \phi \qquad q = 0, 1, 2, \ldots$$

   where ϕ is the net angle contribution at the pole of interest due to all other poles and zeroes of $G(s)H(s)$. The arrival angle at a zero is given by a similar expression:

   $$\phi_z = \pm 180^\circ(2q+1) + \phi \qquad q = 0, 1, 2, \ldots$$

   The angle ϕ is determined by drawing straight lines from all the poles and zeroes to the pole or zero of interest and then summing the angles made by these lines.
7. If a portion of the real axis is part of the root locus and a branch is between two poles, the branch must break away from the real axis so that the locus ends on a zero as k approaches infinity. The breakaway points on the real axis are determined by solving

   $$1 + GH = 0$$

   for k and then finding the roots of the equation $dk/ds = 0$. Only roots that lie on a branch of the locus are of interest.