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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	MATHEMATICS FOR ENGINEERING TECHNOLOGY II
COURSE CODE	:	BDJ 12303
PROGRAMME CODE	:	BDJ
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA CLOSED BOOK 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

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Q1 (a) Solve the following homogeneous equation,

$$\left(x+\sqrt{xy}\right)dy = y\,dx\,.$$

(10 marks)

(b) Given an ordinary differential equation

$$(5x+4y)dx+\left(4x-8y^3\right)dy=0\,.$$

(i) Show that the given equation is exact.

(3 marks)

(ii) Hence, solve the equation for $y(0)=2$.

(12 marks)

Q2 A second order differential equation is defined as

$$y''-2y'+2y=e^x\tan x\,.$$

(a) By assuming that this is a homogeneous equation, solve the equation with initial conditions, $y(0)=y'(0)=0$.

(10 marks)

(b) Find the solution for the given differential equation by using the variation of parameters method.

[Hint: $\int \sec x\,dx=\ln(\sec x\tan x)+c$]

(15 marks)

Q3 (a) Determine the Laplace transform for

(i) $f(t)=4e^{-2t}\sin\frac{3}{2}\pi\cos\frac{3}{2}t.$

(4 marks)

(ii) $f(t)=t^2\sin 2t.$

(5 marks)

(b) Express $\dfrac{1}{(s+1)(s+2)^2}$ in partial fraction form and show that

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)(s+2)^2}\right\} = e^{-t} - (1+t)e^{-2t}.$$

Hence, solve the initial-value problem (IVP)

$$y'' + 4y' + 4y = f(t),\ y(0) = 0 \text{ and } y'(0) = 0,$$

where

$$f(t) = \begin{cases} 0, & 0 \le t < 2, \\ e^{-(t-2)}, & t > 2. \end{cases}$$

(16 marks)

Q4 (a) Consider the following initial-value problem (IVP)

$$(1+x^2)\frac{dy}{dx} - xy = 0,\ y(2) = 5.$$

Solve the IVP for $2 \le x \le 2.2$ and $h = 0.1$ by using fourth-order Runge-Kutta method.

(9 marks)

(b) Given the boundary-value problem (BVP)

$$y'' + 2y' = 2x^2,$$

in the interval [0, 1], with the boundary conditions, $y(0) = 0$ and $y(1) = \dfrac{4}{3}$.

(i) Derive a system of linear equations (in matrix-vector form) using the finite difference method by taking $\Delta x = h = 0.25$.

(12 marks)

(ii) Hence, solve the system.

(4 marks)

– END OF QUESTIONS –

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FORMULA

Second-order Differential Equation

The roots of characteristic equation and the general solution for differential equation $ay'' + by' + cy = 0$.

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i$, $m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

The method of undetermined coefficients

For non-homogeneous second order differential equation $ay'' + by' + cy = f(x)$, the particular solution is given by $y_p(x)$:

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + A_{n-1}x^{n-1} + \cdots + A_1 x + A_0$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}$
$P_n(x)\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)\sin\beta x$
$Ce^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r e^{\alpha x}(P\cos\beta x + Q\sin\beta x)$
$P_n(x)e^{\alpha x}\begin{cases}\cos\beta x \\ \sin\beta x\end{cases}$	$x^r(B_n x^n + B_{n-1}x^{n-1} + \cdots + B_1 x + B_0)e^{\alpha x}\cos\beta x +$ $x^r(C_n x^n + C_{n-1}x^{n-1} + \cdots + C_1 x + C_0)e^{\alpha x}\sin\beta x$

Note : r is the least non-negative integer ($r = 0$, 1, or 2) which determine such that there is no terms in particular integral $y_p(x)$ corresponds to the complementary function $y_c(x)$.

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The method of variation of parameters

If the solution of the homogeneous equation $ay''+by'+cy=0$ is $y_c = Ay_1 + By_2$, then the particular solution for $ay''+by'+cy=f(x)$ is

$$y = uy_1 + vy_2,$$

where $u = -\int \frac{y_2 f(x)}{aW}dx + A,\ v = \int \frac{y_1 f(x)}{aW}dx + B$ and $W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'$

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
e^{at}	$\frac{1}{s-a}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$t^n,\ n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y'(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$		

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Fourth-order Runge-Kutta method

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

which $k_1 = hf(x_i, y_i)$, $k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $k_4 = hf(x_i + h, y_i + k_3)$

Finite difference method

$$y_i' = \frac{y_{i+1} - y_{i-1}}{2h},$$

$$y_i'' = \frac{y_{i+1} - 2y_i + y_{i-1}}{h^2}.$$