UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	: SIGNALS AND SYSTEMS
COURSE CODE	: BEJ 20203
PROGRAMME CODE	: BEJ
EXAMINATION DATE	: JULY/ AUGUST 2023
DURATION	: 3 HOURS
INSTRUCTION	: 1. ANSWER **ALL** QUESTIONS IN **SECTION A** AND **TWO (2)** QUESTIONS ONLY IN **SECTION B**. 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **THIRTEEN (13)** PAGES

SECTION A: ANSWER ALL QUESTIONS.

Q1 (a) Calculate the periodicity of $w(t)$

$$w(t) = 4\cos(4\pi t) + 6\cos(6\pi t) + 2\cos\left(\frac{1}{2}\pi t\right)$$

(7 marks)

(b) Determine whether $x(t)$ as shown in **Figure Q1 (b)** is a power or energy signal. Then, calculate value of power or energy for $x(t)$

(5 marks)

Q2 (a) By using convolution integral, find the overall impulse response $H(t)$ for two cascaded systems with the system impulse responses, $h_1(t)$ and $h_2(t)$ expressed as:

$$h_1(t) = 7e^{-2t}u(t)$$
$$h_2(t) = 3e^{-5t}u(t)$$

(5 marks)

(b) State the commutative property of convolution. Then, show that the answer obtained in **Q2 (a)** satisfies the commutative property of convolution.

(7 marks)

Q3 (a) Explain the Gibbs phenomenon.

(3 marks)

(b) Consider a periodic signal $g(t)$ as shown in **Figure Q3 (a)**. Find the trigonometric Fourier series.

(9 marks)

Q4 (a) Determine the Fourier transform of signal $y(t)$ as shown in **Figure Q4 (a)** by using definition of Fourier Transform.

(4 marks)

(b) Given $x(t) = rect(t)$ and $X(\omega) = sinc\left(\frac{\omega}{2}\right)$. Using properties of Fourier transform, determine Fourier transform of the following signals:

(i) $x(2t)$

(2 marks)

(ii) $x(t-2)$

(2 marks)

(iii) $x(-2(t-2))$

(4 marks)

Q5 (a) Using the definition, find the Laplace transform and its region of convergence (ROC) of $\sin \omega t\,, t > 0$.

(4 marks)

(b) A decaying sinusoidal is given as $x(t) = Ae^{at} \sin \omega t$ for $t > 0$ and $a < 0$. Determine the Laplace transform $X(s)$ using the multiplication by e^{at} property (shift in s-domain).

(2 marks)

(c) Given the Laplace transform of $\sin \omega t$ is

$$L[\sin \omega t] = \frac{\omega}{s^2 + \omega^2}, \qquad Re\{s\} > 0,$$

Determine the Laplace transform of $\cos \omega t$ using the derivative property.

(3 marks)

(d) Given

$$\frac{dy(t)}{dt} + 2y(t) = e^{-t}, \qquad t \geq 0, \qquad y(0) = 1.$$

Find $Y(s)$.

(3 marks)

SECTION B: ANSWER TWO (2) QUESTIONS ONLY.

Q6 (a) A certain communication system produces an even periodic rectangular pulse train signal, $x(t)$ with amplitude of 1V, fundamental frequency $f_0 = 100$ kHz and duty cycle of 40 % ($\tau = 0.4$).

(i) Sketch the signal, $x(t)$.

(2 marks)

(ii) Show that the Fourier series coefficients of the signal is given by

$$x_n = \begin{cases} \frac{2}{5}\text{sinc}\left(\frac{2n}{5}\right) & \text{for } n \neq 0, \\ \frac{2}{5} & \text{for } n = 0. \end{cases}$$

(6 marks)

(iii) Sketch the amplitude spectrum of the Fourier series of $x(t)$ for the first **FIVE (5)** harmonics.

(2 marks)

(b) The signal in **Q6 (a)** is intended for transmission over a transmission media. However, due to limited frequency resources, the channel for the transmission is limited to 1000 kHz. As such, the signal is passed through a simple RC low pass filter with its frequency response given by

$$H(f) = \frac{1}{j2\pi fRC + 1}$$

where RC is the time constant given by

$$RC = \frac{1}{2\pi f_c}$$

and f_c is the cut-off frequency of the filter.

(i) Find the frequency response of the system for $f = 200$ kHz, 400 kHz, 600 kHz, 800 kHz, and 1000 kHz.

(5 marks)

(ii) Evaluate the output of the filter, $y(t)$ for the input signal, $x(t)$ given in **Q6 (a)**.

(5 marks)

Q7 (a) A continuous Linear Time-Invariant (LTI) system is modelled in differentiation equation as follows:

$$\frac{d^2y(t)}{dt^2} + 2\frac{dy(t)}{dt} + y(t) = 4\frac{dx(t)}{dt} + 2x(t)$$

Using Fourier Transform, determine the system function, $H(\omega)$ and the impulse response, $h(t)$ of the system.

(11 marks)

(b) **Figure Q7 (b)** shows an LTI system with the impulse response, $h(t) = e^{-2t}u(t)$ and the input signal, $x(t) = te^{-4t}u(t)$. Determine output of the LTI system $y(t)$ using Fourier Transform.

(9 marks)

Q8 (a) Analyze the system function and impulse response of a stable system for a Linear Time-Invariant (LTI) system given by the differential equation

$$y''(t) - y'(t) - 2y(t) = x'(t) - x(t).$$

(10 marks)

(b) The Laplace transform of a system is given as

$$H(s) = \frac{s^2 - 2s - 11}{s^3 + 4s^2 + s - 6}, \qquad -2 < Re\{s\} < 1.$$

Given the root, $s_{1,2,3} = 1, -2, -3$, analyse the impulse response $h(t)$ of the system with regards to its causality and stability.

(10 marks)

-END OF QUESTIONS –

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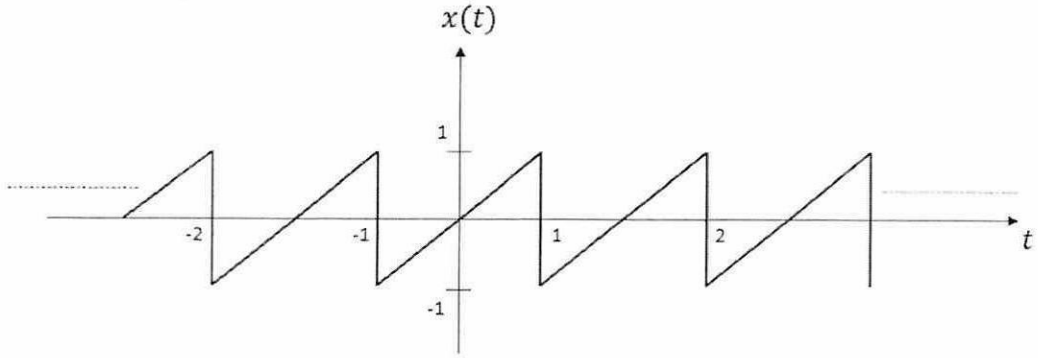

Figure Q1 (b)

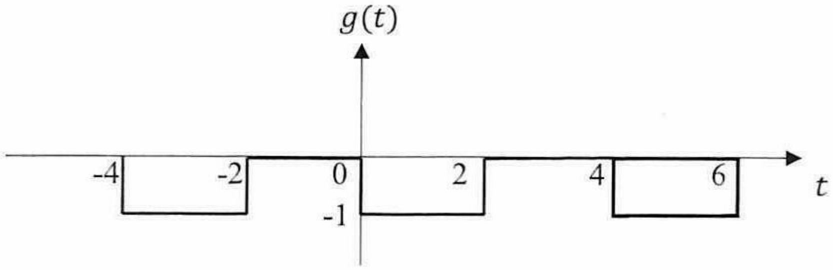

Figure Q3 (a)

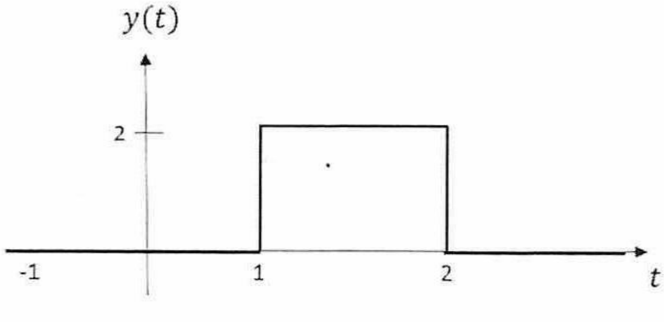

Figure Q4 (a)

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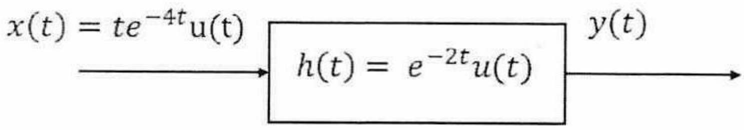

Figure Q7 (b)

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TABLE 1: INDEFINITE INTEGRALS

$\int \cos at\, dt = \frac{1}{a}\sin at$	$\int \sin at\, dt = -\frac{1}{a}\cos at$
$\int t\cos at\, dt = \frac{1}{a^2}\cos at + \frac{1}{a}t\sin at$	$\int t\sin at\, dt = \frac{1}{a^2}\sin at - \frac{1}{a}t\cos at$
$\int te^{at}dt = \frac{1}{a^2}e^{at}(at-1)$	$\int \frac{1}{(a^2+t^2)}dt = \frac{1}{a}\tan^{-1}\left(\frac{t}{a}\right)$

TABLE 2: EULER'S IDENTITY

$e^{\pm j\pi/2} = \pm j$	$A\angle \pm\theta = Ae^{\pm j\theta}$
$e^{\pm jk\pi} = \cos k\pi$	$e^{\pm}j\theta = \cos\theta \pm j\sin\theta$
$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$	$\sin\theta = \frac{1}{2j}\left(e^{j\theta} - e^{-j\theta}\right)$

TABLE 3: COMPLEX NUMBER

$s = a + jb = \|s\|\angle \pm\theta = \|s\|e^{\pm j\theta}$	$\|s\| = \sqrt[2]{a^2+b^2}$	$\theta = \tan^{-1}\left(\frac{b}{a}\right)$

TABLE 4: TRIGONOMETRIC IDENTITIES

$\sin\theta = \cos\left(\theta - \frac{\pi}{2}\right)$	$\cos\theta = \sin\left(\theta + \frac{\pi}{2}\right)$
$\sin(\propto \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$	$\cos(\alpha\pm\beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$
$\sin^2\alpha + \cos^2\beta = 1$	
$\sin 2\alpha = 2\sin\alpha\cos\alpha$	$\cos 2\alpha = 2\cos^2\alpha - 1$
$\cos 2\alpha = 1 - 2\sin^2\alpha$	$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

TABLE 5: VALUES OF COSINE, SINE AND EXPONENTIAL FUNCTIONS FOR INTEGRAL MULTIPLE OF π.

Function	Value	Function	Value
$\cos 2n\pi$	1	$e^{j2n\pi}$	1
$\sin 2n\pi$	0	$e^{jn\pi}$	$(-1)^n$
$\cos n\pi$	$(-1)^n$	$e^{\frac{jn\pi}{2}}$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = \text{even} \\ j(-1)^{\frac{n-1}{2}} & ,n = \text{odd} \end{cases}$
$\sin n\pi$	0		
$\cos\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n}{2}} & ,n = \text{even} \\ 0 & ,n = \text{odd} \end{cases}$	$\sin\left(\frac{n\pi}{2}\right)$	$\begin{cases} (-1)^{\frac{n-1}{2}} & ,n = \text{odd} \\ 0 & ,n = \text{even} \end{cases}$

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TABLE 6: PARTIAL FRACTION FORMULA

Type of proper rational function	Partial Fraction
$\frac{px+q}{(x-a)(x-b)}, a \neq b$	$\frac{A}{x-a}+\frac{B}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x-b)(x-c)}, a \neq b \neq c$	$\frac{A}{x-a}+\frac{B}{x-b}+\frac{C}{x-c}$
$\frac{px+q}{(x-a)^3}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{(x-a)^3}$
$\frac{px^2+qx+r}{(x-a)^2(x-b)}$	$\frac{A}{x-a}+\frac{B}{(x-a)^2}+\frac{C}{x-b}$
$\frac{px^2+qx+r}{(x-a)(x^2+bx+c)}$, where x^2+bx+c cannot be factorised.	$\frac{A}{x-a}+\frac{Bx+C}{x^2+bx+c}$
$\frac{px^3+qx^2+rx+s}{(x^2+ax+b)(x^2+cx+d)}$, where (x^2+ax+b) and (x^2+cx+d) cannot be factorised.	$\frac{Ax+B}{x^2+ax+b}+\frac{Cx+D}{x^2+cx+d}$

TABLE 7: FOURIER SERIES

Exponential	$x(t)=\sum_{n=-\infty}^{\infty} x_n e^{jn\frac{2\pi}{T}t}$ $x_n=\frac{1}{T}\int_{\alpha}^{\alpha+T} x(t)\, e^{-jn\frac{2\pi}{T}t}dt$
Trigonometric	$x(t)=\frac{a_0}{2}+\sum_{n=1}^{\infty}\left(a_n \cos n\frac{2\pi}{T}t+b_n \sin n\frac{2\pi}{T}t\right)$ $a_n=\frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\cos n\frac{2\pi}{T}t\,dt, \quad n=0,1,2,3\ldots$ $b_n=\frac{2}{T}\int_{\alpha}^{\alpha+T} x(t)\sin n\frac{2\pi}{T}t\,dt, \quad n=1,2,3\ldots$
Amplitude-phase	$x(t)=X_0+\sum_{n=1}^{\infty} A_n \cos(n\frac{2\pi}{T}t+\theta_n)$ $A_n=2\lvert X_n\rvert=\sqrt[2]{a_n^2+b_n^2} \quad , \quad \theta_n=\angle X_n=-\tan^{-1}\left(\frac{b_n}{a_n}\right)$
Average Power	$P=V_{dc}I_{dc}+\frac{1}{2}\sum_{n=1}^{\infty} V_n I_n \cos\left(\theta_{V_n}-\theta_{I_n}\right)$

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TABLE 8: DEFINITION OF FOURIER AND LAPLACE TRANSFORM

FOURIER TRANSFORM	INVERSE FOURIER TRANSFORM
$\mathcal{F}[x(t)] = X(\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$ $\mathcal{F}[x(t)] = X(f) = \int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$	$x(t) = \mathcal{F}^{-1}[X(\omega)] = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$ $x(t) = \mathcal{F}^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f)e^{j2\pi ft}\,df$
LAPLACE TRANSFORM	**INVERSE LAPLACE TRANSFORM**
Bilateral $\mathrm{L}[x(t)] = X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$ **Unilateral** $\mathrm{L}[x(t)] = X(s) = \int_{0}^{\infty} x(t)e^{-st}dt$ $s = \sigma + j\omega$	$x(t) = \mathrm{L}^{-1}[X(s)] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}ds$

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TABLE 9: FOURIER TRANSFORM PAIRS

Time domain, $x(t)$	Frequency domain, $X(\omega)$	Frequency domain, $X(f)$
$\delta(t)$	1	1
1	$2\pi\delta(\omega)$	$\delta(f)$
$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	$\frac{\delta(f)}{2} + \frac{1}{j2\pi f}$
$u(t+\tau) - u(t-\tau)$	$\frac{2\sin(\omega\tau)}{\omega} = 2\tau\,\mathrm{sinc}(\omega\tau)$	$2\tau\,\mathrm{sinc}\,2f\tau$
$\mathrm{rect}(t)$	$\mathrm{sinc}\left(\frac{\omega}{2}\right)$	$\mathrm{sinc}(f)$
$\lvert t\rvert$	$-\frac{2}{\omega^2}$	$-\frac{2}{(2\pi f)^2}$
$\mathrm{sgn}(t)$	$\frac{2}{j\omega}$	$\frac{1}{j\pi f}$
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$\frac{1}{\alpha + j2\pi f}$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$\frac{1}{\alpha - j2\pi f}$
$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	$\frac{2a}{a^2+4\pi^2 f^2}$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	$\delta(f-f_0)$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$\frac{n!}{(a+j2\pi f)^{n+1}}$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)]$	$\frac{\delta(f-f_o) - \delta(f+f_o)}{2j}$
$\cos\omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$	$\frac{\delta(f-f_o) + \delta(f+f_o)}{2}$
$e^{-at}\sin\omega_0 t\,u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$\frac{2\pi f_0}{(a+j2\pi f)^2+(2\pi f_0)^2}$
$e^{-at}\cos\omega_0 t\,u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$\frac{a+2\pi f}{(a+j2\pi f)^2+(2\pi f_0)^2}$

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TABLE 10: FOURIER TRANSFORM PROPERTIES

Property	**Time domain,** $x(t)$	**Frequency domain,** $X(\omega)$	**Frequency domain,** $X(f)$
Linearity	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$	$a_1X_1(f) + a_2X_2(f)$
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert}X\left(\frac{f}{a}\right)$
Time shifting	$x(t-t_0)u(t-t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-j2\pi f t_0}X(f)$
Frequency shifting	$e^{j\omega_0 t}x(t)$	$X(\omega-\omega_0)$	$X(f-f_o)$
Modulation	$\cos(\omega_0 t)\,x(t)$ $\sin(\omega_0 t)\,x(t)$	$\frac{1}{2}[X(\omega+\omega_0)+X(\omega-\omega_0)]$ $\frac{1}{2j}[X(\omega-\omega_0)-X(\omega+\omega_0)]$	$\frac{1}{2}[X(f-f_o)+X(f+f_o)]$ $\frac{1}{2j}[X(f-f_o)-X(f+f_o)]$
Time differentiation	$\frac{d}{dt}(x(t))$ $\frac{d^n}{dt^n}(x(t))$	$j\omega X(\omega)$ $(j\omega)^n X(\omega)$	$j2\pi f X(f)$ $(j2\pi f)^n X(f)$
Time integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{X(\omega)}{j\omega} + \pi X(\omega)\,\delta(\omega)$	$\frac{X(f)}{j2\pi f} + \frac{1}{2}X(0)\delta(f)$
Frequency differentiation	$t^n x(t)$	$(j)^n \frac{d^n}{d\omega^n}X(\omega)$	$\left(\frac{j}{2\pi}\right)^n \frac{d^n}{df^n}X(f)$
Time Reversal	$x(-t)$	$X(-\omega)\ or\ X^*(\omega)$	$X(-f)$
Duality	$X(t)$	$2\pi x(-\omega)$	$X(-f)$
Convolution in t	$x_1(t) * x_2(t)$	$X_1(\omega)\cdot X_2(\omega)$	$X(f)\cdot Y(f)$
Multiplication	$x_1(t)\cdot x_2(t)$	$\frac{1}{2\pi}X_1(\omega) * X_2(\omega)$	$X(f) * Y(f)$
Parseval's Theorem	$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2 dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(\omega)\rvert^2 d\omega$	$\int_{-\infty}^{\infty}\lvert X(f)\rvert^2 df$

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TABLE 11: LAPLACE TRANSFORM PAIR

Time domain, $x(t)$, $t > 0$	s-domain, $X(s)$	ROC	Time domain, $x(t)$, $t > 0$	s-domain, $X(s)$	ROC
$\delta(t)$	1	All s	$\cos bt$	$\frac{s}{s^2+b^2}$	$Re(s) > 0$
$u(t)$	$\frac{1}{s}$	$Re(s) > 0$	$\sin bt$	$\frac{b}{s^2+b^2}$	$Re(s) > 0$
t	$\frac{1}{s^2}$	$Re(s) > 0$	$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$Re(s) > -a$
t^n	$\frac{n!}{s^{n+1}}$	$Re(s) > 0$	$e^{-at}\sin bt$	$\frac{b}{(s+a)^2+b^2}$	$Re(s) > -a$
e^{-at}	$\frac{1}{s+a}$	$Re(s) > -a$	$t\cos bt$	$\frac{s^2-b^2}{(s^2+b^2)^2}$	$Re(s) > 0$
te^{-at}	$\frac{1}{(s+a)^2}$	$Re(s) > -a$	$t\sin bt$	$\frac{2bs}{(s^2+b^2)^2}$	$Re(s) > 0$

TABLE 12: LAPLACE TRANSFORM PROPERTIES

Property	Signal	Laplace Transform	ROC
	$x(t)$	$X(s)$	R
	$x_1(t), x_2(t)$	$X_1(s), X_2(s)$	R_1, R_2
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	R
Shifting in the s-Domain	$e^{s_0 t}x(t)$	$X(s - s_0)$	Shifted version of R (i.e., s is in the ROC if $s - s_0$ is in R)
Time scaling	$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{s}{a}\right)$	Scaled ROC (i.e., s is in the ROC if s/a is in R)
Conjugation	$x^*(t)$	$X^*(s^*)$	R
Convolution	$x_1(t) * x_2(t)$	$X_1(s) \cdot X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least R
		$sX(s) - x(0^+)$ (Unilateral)	R right hand plane
	$\frac{d^n}{dt^n}x(t)$	$s^nX(s) - s^{n-1}x(0^+) - \cdots - sx^{n-2}(0^+) - x^{n-1}(0^+)$	
Differentiation in the s-Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	R
Integration in the Time Domain	$\int_{-\infty}^{t} x(\tau)d\tau$	$\frac{1}{s}X(s)$	At least $R \cap \{Re(s) > 0\}$

Initial- and Final- Value Theorems

If $x(t) = 0$ for $t < 0$ and $x(t)$ contains no impulses or higher order singularities at $t = 0$, then

$$x(0^+) = \lim_{s\to\infty} sX(s)$$

If $x(t) = 0$ for $t < 0$ and has a finite limit as $t \to \infty$, then

$$\lim_{t\to\infty} x(t) = \lim_{s\to\infty} sX(s)$$