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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2014/2015

COURSE NAME	:	ENGINEERING MATHEMATICS I
COURSE CODE	:	DAS 10203
PROGRAMME	:	1 DAA / 1 DAM
EXAMINATION DATE	:	JUNE 2015 / JULY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN **PART A** B) ANSWER **THREE (3)** QUESTIONS ONLY IN **PART B**

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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PART A

Q1 (a) Evaluate

(i) $\int \left(2x^5 + 3\sqrt[3]{x} - \frac{4}{x^4} \right) dx$

(2 marks)

(ii) $\int \left(e^{2x} + \cos \pi x \right) dx$

(2 marks)

(iii) $\int_0^4 \left(\frac{2\sqrt{x} + x^4}{x^2} \right) dx$

(4 marks)

(b) Solve the integration using the given method.

(i) $\int \frac{4x+6}{\left(x^2+3x+7\right)^4} dx$ (substitution)

(4 marks)

(ii) $\int x \ln x \, dx$ (part by part)

(4 marks)

(iii) $\int x^2 e^{2x} \, dx$ (tabular)

(4 marks)

Q2 (a) Curve $y = x^2 - 1$ and line $y = 2x + 7$ crosses at points A and B.

(i) Determine points A and B.

(4 marks)

(ii) Sketch the graph

(2 marks)

(iii) Find the area enclosed by the curve and the line.

(4 marks)

(b) Use cylindrical shells method to find the volume of the solid that results when the region enclosed by $y = 2x - x^2$ and $y = 0$ is revolved about the y – axis between $x = 0$ and $x = 2$.

(3 marks)

(c) Calculate the arc length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 5$.

(7 marks)

PART B

Q3 (a) The function $f(x)$ is given by

$$f(x)=\begin{cases}4 & x\le -2\\ x^2 & -2<x<2\\ x+2 & x\ge 2\end{cases}$$

(i) Sketch the graph of $f(x)$.

(3 marks)

(ii) Write the domain and range for $f(x)$.

(2 marks)

(iii) Calculate the value of $f(x)$ when $x=-4$, $x=1$ and $x=6$.

(3 marks)

(b) Given $f(x)=e^x+1$, $g(x)=\ln x$, and $h(x)=x-1$. Find the composite function of

(i) $f\circ g(1)$

(3 marks)

(ii) $g\circ h\circ f(x)$

(4 marks)

(c) If $h(x)=cx+2$, find

(i) the inverse function, $h^{-1}(x)$.

(3 marks)

(ii) c if $h^{-1}(3)=\dfrac{1}{5}$.

(2 marks)

Q4 (a) Let

$$f(x)=\begin{cases} x & \text{if } x \le -1 \\ x^2+3 & \text{if } -1 < x < 3 \\ 12 & \text{if } x \ge 3 \end{cases}$$

Compute the following limits or state that they do not exist.

(i) $\lim_{x \to -1^-} f(x)$, $\lim_{x \to -1^+} f(x)$, and $\lim_{x \to -1} f(x)$

(4 marks)

(ii) $\lim_{x \to 2^-} f(x)$, $\lim_{x \to 2^+} f(x)$, and $\lim_{x \to 2} f(x)$

(4 marks)

(b) Evaluate the following limits

(i) $\lim_{x \to \infty} \left(2 - \frac{1}{x} + \frac{10}{x^2} \right)$

(3 marks)

(ii) $\lim_{x \to \infty} \frac{3x+2}{5-4x}$

(3 marks)

(c) Discuss whether the function given is continuous at $x = 2$.

$$g(x)=\begin{cases} \dfrac{x^2-4}{x-2} & \text{if } x \ne 2 \\ 4 & \text{if } x = 2 \end{cases}$$

(6 marks)

Q5 (a) Find $\frac{dy}{dx}$ of

(i) $y(x) = \frac{e^{-2x}}{1+x^2}$

(4 marks)

(ii) $\cos 2x + 2x^2 y = 6 - y^2$

(4 marks)

(iii) $y(x) = \ln(2 - x^2)$

(3 marks)

(b) Using partial fraction method, evaluate

$$\int \left(\frac{3x+7}{x^2 - 2x - 3} \right) dx$$

(9 marks)

Q6 (a) Using L'Hospital's Rule , find

(i) $\lim_{x \to \infty} \frac{4x^2 - 5x}{7 - 3x^2}$

(4 marks)

(ii) $\lim_{x \to 0} \frac{\sin x}{x}$

(3 marks)

(iii) $\lim_{x \to \infty} \frac{e^x}{3x^2}$

(3 marks)

(b) Determine the absolute extrema for the function $f(x) = \frac{x^4}{4} + x^3 - \frac{x^2}{2} - 3x + 5$ on the interval $[-5, 2]$.

(10 marks)

Q7 (a) Find the approximate value for $\int_0^1 \sqrt{x^2+1}\,dx$ using trapezoidal rule by taking step size $h = 0.2$.

(8 marks)

(b) Find the approximate value for $\int_1^3 \frac{1}{x+1}\,dx$ using $\frac{1}{3}$ Simpson's Rule by taking $n = 8$ subintervals.

(8 marks)

(c) A rectangular water tank is being filled at the constant rate of 2000 liter/sec. The base of the tank has dimension width = 100 cm and length 250 cm. Find the rate of change of height of water in the tank, given the volume, $V = w \times l \times h$ where w = width, l = length and h =height.

(4 marks)

– END OF QUESTION –

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Formula

Table 1 : Differentiation

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\cos x]=-\sin x$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\tan x]=\sec^2 x$
$\frac{d}{dx}[cu]=c\frac{du}{dx}$	$\frac{d}{dx}[\sec x]=\sec x\tan x$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}[\cot x]=-\csc^2 x$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[\csc x]=-\csc x\cot x$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\sin^{-1} x]=\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}[\ln\lvert x\rvert]=\frac{1}{x}$	$\frac{d}{dx}[\cos^{-1} x]=\frac{-1}{\sqrt{1-x^2}}$
$\frac{d}{dx}\left[e^{nx}\right]=ne^{nx}$	$\frac{d}{dx}\left[\tan^{-1} x\right]=\frac{1}{1+x^2}$
$\frac{d}{dx}[\sin x]=\cos x$	$\frac{d}{dx}\left[\sec^{-1} x\right]=\frac{1}{\lvert x\rvert\sqrt{x^2-1}}$

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Table 2 : Integration

$\int c\, f(x)dx = c\, F(x) + C$	$\int \tan x\, dx = \ln\lvert \sec x \rvert + C$
$\int [f(x) \pm g(x)]dx = F(x) \pm G(x) + C$	$\int \sec^2 x\, dx = \tan x + C$
$\int x^r\, dx = \frac{x^{r+1}}{r+1} + C, (r \neq -1)$	$\int \csc^2 x\, dx = -\cot x + C$
$\int u dv = uv - \int v du$	$\int \csc x\, dx = -\ln\lvert \csc x + \cot x \rvert + C$
$\int \cos nx\, dx = \frac{1}{n}(\sin nx) + C$	$\int \sec x \tan x\, dx = \sec x + C$
$\int \sin nx\, dx = -\frac{1}{n}(\cos nx) + C$	$\int \csc x \cot x\, dx = -\csc x + C$
$\int e^{nx}\, dx = \frac{1}{n}\left(e^{nx}\right) + C$	$\int \sec x\, dx = \ln\lvert \sec x + \tan x \rvert + C$
$\int \frac{1}{nx+b}\, dx = \frac{1}{n}\ln\lvert nx+b \rvert + C$	$\int \frac{1}{b-nx}\, dx = -\frac{1}{n}\ln\lvert b-nx \rvert + C$

Area of Region

$$A = \int_a^b [f(x) - g(x)]dx \quad \text{or} \quad A = \int_c^d [w(y) - v(y)]dy$$

Volume Cylindrical Shells

$$V = \int_a^b 2\pi x\, f(x)\, dx \quad \text{or} \quad V = \int_c^d 2\pi y\, f(y)\, dy$$

Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{or} \quad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

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Area of Surface

$$S=\int_a^b 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad \text{or} \quad S=\int_c^d 2\pi x\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

Partial Fraction

$$\frac{a}{(s+b)(s^2+c)}=\frac{A}{(s+b)}+\frac{Bs+C}{(s^2+c)}$$

$$\frac{a}{s(s-b)(s-c)}=\frac{A}{s}+\frac{B}{s-b}+\frac{C}{s-c}$$

$$\frac{a}{(s+b)(s-c)}=\frac{A}{(s+b)}+\frac{B}{(s-c)}$$

Simpson's Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[(f_0+f_n)+4\sum_{\substack{i=1\\ i\text{ odd}}}^{n-1} f_i+2\sum_{\substack{i=2\\ i\text{ even}}}^{n-2} f_i\right]; \quad n=\frac{b-a}{h}$$

Trapezoidal Rule

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[(f_0+f_n)+2\sum_{i=1}^{n-1} f_i\right]; \quad n=\frac{b-a}{h}$$