Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	CALCULUS
COURSE CODE	:	BEE 10103
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JULY/ AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Determine the gradient of the tangent at a point (-2, 1) to a graph of

$$xy^2 + 3x = \frac{x}{y^2} - 6$$

(5 marks)

(b) Find derivatives for the following functions.

(i) $x = t^2 + 2$, $y = sin(t + 1)$

(4 marks)

(ii) $y = \sin^4 3x$

(4 marks)

(iii) $y = \tanh(2x + 3)$

(4 marks)

(iv) $y = \ln \frac{(x^2+1)}{\sqrt{x^3}}$

(4 marks)

(c) Find the limit of $\lim_{x\to\infty} xe^x$ by using L'Hopital's rule.

(4 marks)

Q2 (a) Evaluate the following integrals.

(i) $\int \frac{dx}{x^3}$

(2 marks)

(ii) $\int (3x^2 + 1)e^{(x^3+x)}\ dx$

(2 marks)

(iii) $\int_{-2}^{3} |2x - 5|\ dx$

(5 marks)

(b) Compute $\int \frac{2x+1}{(x+2)^2(x+1)}$ dx using partial fraction method.

(6 marks)

(c) Evaluate the following functions using integration by parts.

$$\int x^3 \, ln\, x^2 dx$$

(5 marks)

(d) Solve the following integral by using the tabular method.

$$\int e^x \sin x \, dx$$

(5 marks)

Q3 (a) Evaluate each of the following integrals.

(i) $\int sin^5 x \; dx$

(5 marks)

(ii) $\int x^3 \, sinh\, x \; dx$

(4 marks)

(b) For each of the functions below find $f^{-1}(x)$.

(i) $f(x) = \frac{1}{\sqrt{x^2+1}}$

(4 marks)

(ii) $f(x) = e^{2x+1}$

(4 marks)

(c) Find the derivative of inverse function for the following functions.

(i) $f(x) = \sqrt{x^2+1}$

(4 marks)

(i) $f(x) = \tan x$

(4 marks)

Q4 (a) Given $\cosh^{-1} y + \sinh x = 1$, prove that $\frac{dy}{dx} = -\frac{\sqrt{x^2-1}}{\sqrt{1+x^2}}$.

(3 marks)

(b) Solve $\int \frac{1}{\sqrt{2x-x^2}} \, dx$.

(5 marks)

(c) Find the differentiation of the function $y = \frac{\tan^{-1}(2x)}{\cot^{-1} x^2}$.

(8 marks)

(d) Evaluate the following integrals.

(i) $\int \frac{1}{x^2+8x+17} dx$, by using inverse trigonometric function.

(4 marks)

(ii) $\int \frac{x}{\sqrt{x^2+9}} dx$, by using hyperbolic substitution.

(5 marks)

-END OF QUESTIONS-

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FORMULAE

Indefinite Integrals	Integration Of Inverse Functions
$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{1}{x} dx = \ln\|x\| + C$	$\int \frac{-1}{\sqrt{a^2 - x^2}} dx = \cos^{-1}\left(\frac{x}{a}\right) + C$
$\int \cos x \, dx = \sin x + C$	$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \sin x \, dx = -\cos x + C$	$\int \frac{-1}{a^2 + x^2} dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec^2 x \, dx = \tan x + C$	$\int \frac{1}{\|x\|\sqrt{x^2 - a^2}} dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc^2 x \, dx = -\cot x + C$	$\int \frac{-1}{\|x\|\sqrt{x^2 - a^2}} dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$
$\int \sec x \tan x \, dx = \sec x + C$	$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$
$\int \csc x \cot x \, dx = -\csc x + C$	$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$
$\int e^x dx = e^x + C$	$\int \frac{-1}{\|x\|\sqrt{a^2 - x^2}} dx = \frac{1}{a}\text{sech}^{-1}\left\|\frac{x}{a}\right\| + C$
$\int \cosh x \, dx = \sinh x + C$	$\int \frac{-1}{\|x\|\sqrt{a^2 + x^2}} dx = \frac{1}{a}\text{csch}^{-1}\left\|\frac{x}{a}\right\| + C$
$\int \sinh x \, dx = \cosh x + C$	$\int \frac{1}{a^2 - x^2} dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \|x\| < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \|x\| > a \end{cases}$
$\int \text{sech}^2 x \, dx = \tanh x + C$	
$\int \text{csch}^2 x \, dx = -\coth x + C$	
$\int \text{sech}\, x \tanh x \, dx = -\text{sech}\, x + C$	
$\int \text{csch}\, x \coth x \, dx = -\text{csch}\, x + C$	

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TRIGONOMETRIC/ HYPERBOLIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x=k\tan\theta$	$x=k\sinh\theta$
$\sqrt{x^2-k^2}$	$x=k\sec\theta$	$x=k\cosh\theta$
$\sqrt{k^2-x^2}$	$x=k\sin\theta$	$x=k\tanh\theta$

WEIERSTRASS SUBSTITUTION

$\tan\frac{1}{2}x=t$		$\tan x=t$	
$\sin x=\frac{2t}{1+t^2}$ $\tan x=\frac{2t}{1-t^2}$	$\cos x=\frac{1-t^2}{1+t^2}$ $dx=\frac{2dt}{1+t^2}$	$\sin 2x=\frac{2t}{1+t^2}$ $\tan 2x=\frac{2t}{1-t^2}$	$\cos 2x=\frac{1-t^2}{1+t^2}$ $dx=\frac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions

$\cos^2 x+\sin^2 x=1$

$\sin 2x=2\sin x\cos x$

$\cos 2x=\cos^2 x-\sin^2 x$

$=2\cos^2 x-1$

$=1-2\sin^2 x$

$1+\tan^2 x=\sec^2 x$

$1+\cot^2 x=\csc^2 x$

$\tan 2x=\frac{2\tan x}{1-\tan^2 x}$

$\tan(x\pm y)=\frac{\tan x\pm\tan y}{1\mp\tan x\tan y}$

$\sin(x\pm y)=\sin x\cos y\pm\sin y\cos x$

$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$

$2\sin ax\cos bx=\sin(a+b)x+\sin(a-b)x$

$2\sin ax\sin bx=\cos(a-b)x-\cos(a+b)x$

$2\cos ax\cos bx=\cos(a-b)x+\cos(a+b)x$

Hyperbolic Functions

$\sinh x=\frac{e^x-e^{-x}}{2}$

$\cosh x=\frac{e^x+e^{-x}}{2}$

$\cosh^2 x-\sinh^2 x=1$

$\sinh 2x=2\sinh x\cosh x$

$\cosh 2x=\cosh^2 x+\sinh^2 x$

$=2\cosh^2 x-1$

$=1+2\sinh^2 x$

$1-\tanh^2 x=\text{sech}^2 x$

$\coth^2 x-1=\text{csch}^2 x$

$\tanh 2x=\frac{2\tanh x}{1+\tanh^2 x}$

$\tanh(x\pm y)=\frac{\tanh x\pm\tanh y}{1\pm\tanh x\tanh y}$

$\sinh(x\pm y)=\sinh x\cosh y\pm\sinh y\cosh x$

$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$