Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS / ENGINEERING MATHEMATICS II
COURSE CODE	:	BEE 11203 / BEE 11403
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	1	ANSWER **ALL** QUESTIONS.
	2	THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**.
	3	STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

Q1 Prove that $\frac{dy}{dx} = \frac{6x^2y+x^3}{-2x^3}$ with an initial condition of $y(1) = 0$ has a particular solution as below, given that the differential equation is either linear or homogenous:

$$8x^3y + x^4 = 1$$

(a) Linear equation; or

(12 marks)

(b) Homogeneous equation.

(13 marks)

Q2 Given a second-order ODE, $\frac{d^2y}{dx^2} = y + xe^x$:

(a) Identify the complementary function, y_c for the corresponding homogeneous equation.

(2 marks)

(b) Solve the given second-order ODE by using the undetermined coefficient method.

(10 marks)

(c) Then, show that the result will be the same as in **Q2(b)** by using the variation of parameter method.

(13 marks)

Q3 (a) A system of first-order differential equation consists of:

$$y_1' = 3y_1 + 5y_2, \text{ and } y_2' = -y_1 - 3y_2.$$

(i) Write the equations for the system in matrix form, $\mathbf{Y}'$.

(1 mark)

(ii) Evaluate the eigenvalues, λ_1 and λ_2 for the system, and the corresponding eigenvectors, $\boldsymbol{V_1}$ and $\boldsymbol{V}_2$.

(6 marks)

(iii) Find the general solutions, y_1 and y_2, for the system.

(3 marks)

(b) A system of first-order differential equation is given as $y_1' = 5y_1 + 4y_2 - 5x^2 + 6x + 25$, and $y_2' = y_1 + 2y_2 - x^2 + 2x + 4$. The eigenvalues for the system are $\lambda_1 = 1$ and $\lambda_2 = 6$, and the corresponding eigenvectors are $\boldsymbol{V}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\boldsymbol{V}_2 = \begin{pmatrix} 1 \\ 1/_4 \end{pmatrix}$. The eigenvectors are linearly independent.

(i) Write the general solutions, $\boldsymbol{Y}_c$ for the homogenous system.

(1 mark)

(ii) Determine the particular integral, $\mathbf{Y}_p$ for the non-homogeneous system using the undetermined coefficients method.

(9 marks)

(iii) Obtain the general solution of the non-homogeneous system.

(1 mark)

(iv) Compute the particular solutions, y_1 and y_2, given that the initial conditions are $y_1(0) = y_2(0) = 0$.

(4 marks)

Q4 (a) Given an RC circuit as shown in **Figure Q4(a)** with $R = 10^6\ \Omega, C = 1\ \mu\text{F}$, $E(t) = 10\ V$. At time $t = 2$ seconds, the switch is thrown from position Q to P for 1 second before switching back to Q.

(i) Show that the circuit can be modelled as:

$$RC\frac{dV}{dt} + V = 10\left[H(t-2) - H(t-3)\right]$$

(3 marks)

(ii) Then calculate the response, $V(t)$ with $V(0) = 0$ using Laplace transform.

(8 marks)

(b) A circuit in series has two identical electromotive forces with each represented by V, a resistor of $2\ \Omega$, an inductor of 0.1 H, and a capacitor of 0.5 F (See **Figure Q4(b)**). At time, $t \leq 0$ second, there is no current flows through the circuit. At time interval, $0 < t \leq 5$ seconds, terminal P is connected to terminal B. After 5 seconds, terminal P is switched to terminal A. Assume $i(t)$ is the current across the circuit at time t.

(i) Write down the initial condition of the circuit in **Figure Q4(b)**.

(1 mark)

(ii) Apply the Kirchhoff's Voltage Law (KVL) for the circuit in **Figure Q4(b)** and show that the circuit above can be modelled as:

$$\frac{d\,i(t)}{dt} + 20i(t) + 20\int_0^t i(\tau)\,d\tau = 2 - H(t-5)$$

(3 marks)

(iii) Find the general solution for $i(t)$.

(10 marks)

- END OF QUESTIONS –

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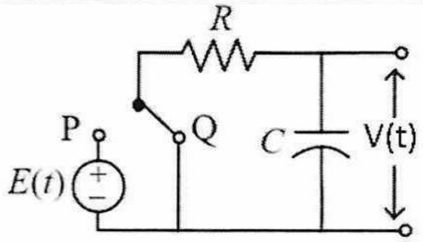

Figure Q4(a)

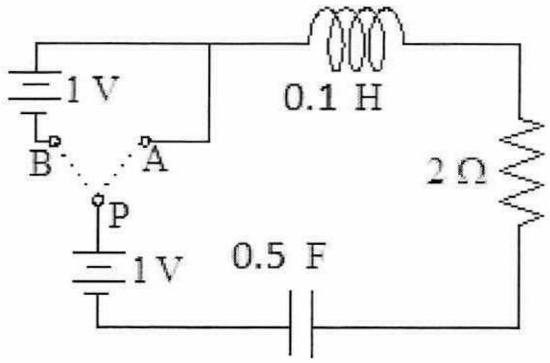

Figure Q4(b)

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First Order Homogeneous Differential Equation

$$\frac{dy}{dx} = f(x, y)$$

$$y = vx, \quad \frac{dy}{dx} = v + x\frac{dv}{dx}$$

First Order Linear Differential Equation

$$y' + p(x)y = q(x)$$

The solution is given by $e^{\int p(x)dx} y = \int e^{\int p(x)dx} q(x) + C$

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SECOND-ORDER LINEAR CONSTANT COEFFICIENT HOMOGENEOUS DIFFERENTIAL EQUATION

The roots of characteristic equation and the general solution for differential equation
$$ay''(x) + by'(x) + cy(x) = 0.$$

Characteristic equation: $am^2 + bm + c = 0$.		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 x} + Be^{m_2 x}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bx)e^{mx}$
3.	Complex roots: $m_1 = \alpha + \beta i, m_2 = \alpha - \beta i$	$y = e^{\alpha x}(A \cos \beta x + B \sin \beta x)$

SECOND-ORDER LINEAR CONSTANT COEFFICIENT NONHOMOGENEOUS DIFFERENTIAL EQUATION

$$ay''(x) + by'(x) + cy(x) = f(x)$$

Variation of parameters

$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	
$u = -\int \frac{y_2 f(x)}{aW} dx + A$	$v = \int \frac{y_1 f(x)}{aW} dx + B$
$y = uy_1 + vy_2$	

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The method of undetermined coefficients for second order linear constant coefficient nonhomogeneous differential equations

$$ay''(x) + by'(x) + cy(x) = f(x)$$

General solution is $y = y_c + y_p$

Particular Integral, y_p

Type of $f(x)$	**Example of $f(x)$**	**Assumption of y_p**
Exponent	ke^{nx}	Ce^{nx}
Polynomial	k	C
	kx	$Cx+D$
	kx^2	Cx^2+Dx+E
	kx^n	$C_nx^n+C_{n-1}x^{n-1}+\ldots+C_1x+C_0$
Trigonometry (sin and cos only)	$k\sin nx$ or $k\cos nx$	$y_p = C\cos nx + D\sin nx$
	$k\sinh nx$ or $k\cosh nx$	$y_p = C\cosh nx + D\sinh nx$
Product of polynomial and exponential	$P_n(x)e^{nx}$	$(C_nx^n+C_{n-1}x^{n-1}+\ldots+C_1x+C_0)e^{nx}$
Product of polynomial and trigonometry	$P_n(x)\sin nx$ or $P_n(x)\cos nx$	$(C_nx^n+C_{n-1}x^{n-1}+\ldots+C_1x+C_0)\sin nx +$ $(D_nx^n+D_{n-1}x^{n-1}+\ldots+D_1x+D_0)\cos nx$
Product of exponential and trigonometry	$ke^{nx}\sin nx$	$e^{nx}\left(C\cos nx + D\sin nx\right)$

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Homogeneous System of first-order differential equation

$$\mathbf{Y}'(x) = A\mathbf{Y}(x)$$

Eigenvalues

$|A - \lambda \mathbf{I}| = 0$

Eigenvectors

$(A - \lambda \mathbf{I})\mathbf{V} = 0$

Case	Roots	General solution
1	Real and Distinct eigenvalues	$\mathbf{Y} = A\mathbf{V}_1 e^{\lambda_1 x} + B\mathbf{V}_2 e^{\lambda_2 x}$
2	Repeated eigenvalues	$\mathbf{Y} = A\mathbf{V}_1 e^{\lambda x} + B\left[\mathbf{V}_1 x + \mathbf{V}_2\right] e^{\lambda x}$

Nonhomogeneous system of first-order linear differential equations

$$\mathbf{Y}'(x) = A\,\mathbf{Y}(x) + \mathbf{G}(x)$$

General solution is $\mathbf{Y} = \mathbf{Y}_c + \mathbf{Y}_p$

Particular Integral, $\mathbf{Y}_p$

Assume $\mathbf{Y}_p$ based on $\mathbf{G}$		
Case	$\mathbf{G}(x)$	$\mathbf{Y}_p$
Case I	Polynomial $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$	$\begin{pmatrix} C \\ D \end{pmatrix}$
	$\begin{pmatrix} a_1 x + b_1 \\ a_2 x + b_2 \end{pmatrix}$	$\begin{pmatrix} Cx + E \\ Dx + F \end{pmatrix}$
	$\begin{pmatrix} a_1 x^2 + b_1 x + c_1 \\ a_2 x^2 + b_2 x + c_2 \end{pmatrix}$	$\begin{pmatrix} Cx^2 + Ex + G \\ Dx^2 + Fx + H \end{pmatrix}$
Case II	Exponent $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} e^{kx}$	$\begin{pmatrix} C \\ D \end{pmatrix} e^{kx}$ if $\mathbf{Y}_p \equiv \mathbf{Y}_C$, then $\begin{pmatrix} C \\ D \end{pmatrix} xe^{kx} + \begin{pmatrix} E \\ F \end{pmatrix} e^{kx}$
Case III	Trigonometric (sin and cos only) $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \sin kx$ or $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cos kx$	$\begin{pmatrix} C \\ D \end{pmatrix} \sin kx + \begin{pmatrix} E \\ F \end{pmatrix} \cos kx$

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Electrical Formula

1. Voltage drop across resistor, R (Ohm's Law): $v_R = iR$
2. Voltage drop across inductor, L (Faraday's Law): $v_L = L\dfrac{di}{dt}$
3. Voltage drop across capacitor, C (Coulomb's Law): $v_C = \dfrac{q}{C}$ or $i = C\dfrac{dv_C}{dt}$
4. The relation between current, i and charge, q: $i = \dfrac{dq}{dt}$.

Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\dfrac{a}{s}$	$\int_0^t f(\tau)\,d\tau$	$\dfrac{F(s)}{s}$
e^{at}	$\dfrac{1}{s-a}$	$H(t-a)$	$\dfrac{e^{-as}}{s}$
$\sin at$	$\dfrac{a}{s^2+a^2}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\cos at$	$\dfrac{s}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\sinh at$	$\dfrac{a}{s^2-a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cosh at$	$\dfrac{s}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
t^n, $n=1,2,3,\ldots$	$\dfrac{n!}{s^{n+1}}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$y'(t)$	$sY(s)-y(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \dfrac{d^n}{ds^n}F(s)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$