CONFIDENTIAL

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2022/2023

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|---|---|---|
| COURSE NAME | : | MULTIVARIABLE CALCULUS / ENGINEERING MATHEMATICS III |
| COURSE CODE | : | BEE 20303 / BEE 21503 |
| PROGRAMME CODE | : | BEJ / BEV |
| EXAMINATION DATE | : | JULY/AUGUST 2023 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | 1. | ANSWER ALL QUESTIONS |
| | 2. | THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. |
| | 3. | STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK |

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES



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**Q1** (a) Given $f(x,y,z) = 3x^2 - 2y^3 + z^2$, $x = 2\,t$, $y = e^t$, $z = \sin t$. Use Chain Rule to find $\frac{df}{dt}$.

(6 marks)

(b) Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y)$ is implicitly defined as a function of $x$ and $y$ for $z^2 + ye^{xz} = z\sin xy^2 + 1$.

(6 marks)

(c) Sketch and calculate the area of a region, $R$ enclosed by the followings;

(i) Coordinate (0,0), (2,0) and $(0,\frac{1}{2})$.

(5 marks)

(ii) $y \le x$ and $y = \cos x$ in the first quadrant.

(8 marks)

**Q2** (a) Find the surface area of the part of $z = xy$ that lies in the cylinder $x^2 + y^2 = 16$ in the first quadrant.

(9 marks)

(b) Find the mass of the solid bounded by $z = 4 - x^2 + y^2$ and below by $xy$-plane if the density of the solid is given by $\delta(x,y,z) = 2 + x + y$.

(8 marks)

(c) Find the volume of the solid that lies between a sphere $x^2 + y^2 + z^2 = 9$ and $x^2 + y^2 + z^2 = 25$ in the region $y \ge 0$ and $z \ge 0$.

(8 marks)

**Q3** (a) Verify Green's theorem for the line integral $\oint_C (x-y)dx + (x+y)dy$, where $C$ is the unit square consisting of vertices (0,0), (1,0), (1,1) and (0,1).

(16 marks)

(b) Given vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ and curve $C$ along the line segment from (0,0) to (3,3).

(i) Show that $\mathbf{F}$ is a conservative vector field.

(1 mark)

(ii) Then, evaluate the work done by vector field $\mathbf{F}$ along the curve $C$ using potential function.

(4 marks)

(iii) Compare your answer in **Q3(b)(ii)** using the line integral over the simplest path.

(4 marks)

**Q4** (a) Differentiate between Gauss's theorem and Stokes' theorem.

(2 marks)

(b) Given that σ is the surface of the solid $G$ enclosed by cone $z = 4 - \sqrt{x^2 + y^2}$ and plane $z = 0$.

(i) Compute the flux of water flowing through the cone's and the circle's surfaces if the velocity vector, $\mathbf{F} = 3x\boldsymbol{i} + 3y\mathbf{j} + 6\mathbf{k}$. Assume that the unit normal vector is oriented outward.

(8 marks)

(ii) Evaluate $\iint_{\sigma} \vec{F} \cdot \hat{n} ds$ by using Gauss's theorem.

(4 marks)

(c) Let σ be the portion of paraboloid $z = 4 - x^2 - y^2$, $z \geq 0$, and oriented outward. Suppose that the curve $C$ is the boundary of σ in the $xy$-plane and the force field is given by $\mathbf{F} = 2z\mathbf{i} + 3x\mathbf{j} + 5y\mathbf{k}$.

(i) Calculate the work done by the force field along curve $C$.

(5 marks)

(ii) Verify Stokes' Theorem.

(6 marks)

**-END OF QUESTIONS-**

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## FORMULAS

Polar coordinate

$x = r\cos\theta,\ y = r\sin\theta,\ \theta = \tan^{-1}(y/x)$, and $\iint_R f(x,y)dA = \iint_R f(r,\theta)\, r\, dr\, d\theta$

Cylindrical coordinate

$x = r\cos\theta,\ y = r\sin\theta,\ z = z$ and $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z) r\, dz\, dr\, d\theta$

Spherical coordinate

$x = \rho\sin\phi\cos\theta,\ y = \rho\sin\phi\sin\theta,\ z = \rho\cos\phi$, then $x^2 + y^2 + z^2 = \rho^2$, for $0 \le \theta \le 2\pi,\ 0 \le \phi \le \pi$,

and $\iiint_G f(x,y,z)dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi\, d\rho\, d\phi\, d\theta$

$A = \iint_R dA$

$m = \iint_R \delta(x,y)dA$, where $\delta(x,y)$ is a density of lamina

$V = \iint_R f(x,y)\, dA$

$V = \iiint_G dV$

$m = \iiint_G \delta(x,y,z)dV$

If $f$ is a differentiable function of $x$, $y$ and $z$, then the

**Gradient of $f$**, grad $f(x,y,z) = \nabla f(x,y,z) = \frac{\partial f}{\partial x}\mathbf{i} + \frac{\partial f}{\partial y}\mathbf{j} + \frac{\partial f}{\partial z}\mathbf{k}$

If $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is a vector field in Cartesian coordinate, then the

**Divergence of** $\mathbf{F}(x,y,z)$, div $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$ **Curl of** $\mathbf{F}(x,y,z)$,

$$\text{curl}\,\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial Z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial Z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

$\mathbf{F}$ is a conservative vector field if Curl of $\mathbf{F} = 0$.

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***Surface Integral***

Let $S$ be a surface with equation $z = g(x, y)$ and let $R$ be its projection on the $xy$-plane.

$$\iint_S f(x,y,z)\,dS = \iint_R f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2+\left(\frac{\partial z}{\partial y}\right)^2}\,dA$$

***Gauss's Theorem***

$$\iint_S \mathbf{F}\bullet\mathbf{n}dS = \iiint_G \nabla\bullet\mathbf{F}dV$$

***Stokes' Theorem***

$$\iint_S (\nabla\times\mathbf{F})\bullet\mathbf{n}dS = \oint_C \mathbf{F}\bullet dr$$

***Identities of Trigonometry and Hyperbolic***

Trigonometric Functions

$\cos^2 x+\sin^2 x=1$

$\sin 2x = 2\sin x\cos x$

$\cos 2x = \cos^2 x - \sin^2 x$

$= 2\cos^2 x - 1$

$= 1 - 2\sin^2 x$

$1+\tan^2 x = \sec^2 x$

$1+\cot^2 x = \csc^2 x$

$\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$

$\tan(x\pm y) = \dfrac{\tan x \pm \tan y}{1\mp \tan x\tan y}$

$\sin(x\pm y) = \sin x\cos y \pm \cos x \sin y$

$\cos(x\pm y) = \cos x\cos y \mp \sin x\sin y$

$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$

$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$

$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$

Hyperbolic Functions

$\sinh x = \dfrac{e^x - e^{-x}}{2}$

$\cosh x = \dfrac{e^x + e^{-x}}{2}$

$\cosh^2 x - \sinh^2 x = 1$

$\sinh 2x = 2\sinh x\cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$= 2\cosh^2 x - 1$

$= 1 + 2\sinh^2 x$

$1-\tanh^2 x = \operatorname{sech}^2 x$

$\coth^2 x - 1 = \operatorname{csch}^2 x$

$\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$

$\tanh(x\pm y) = \dfrac{\tanh x \pm \tanh y}{1\pm \tanh x\tanh y}$

$\sinh(x\pm y) = \sinh x\cosh y \pm \cosh x\sinh y$

$\cosh(x\pm y) = \cosh x\cosh y \pm \sinh x\sinh y$

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***The derivative of f(x) with respect to x***

$$f_x(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

***Indefinite Integrals and Integration of Inverse Functions***

Indefinite Integrals

$$\int x^n dx=\frac{x^{n+1}}{n+1}+C,\qquad n\neq -1$$

$$\int \frac{1}{x}dx=\ln|x|+C$$

$$\int \cos x dx=\sin x+C$$

$$\int \sin x dx=-\cos x+C$$

$$\int \sec^2 x dx=\tan x+C$$

$$\int \csc^2 x dx=-\cot x+C$$

$$\int \sec x\tan x dx=\sec x+C$$

$$\int \csc x\cot x dx=-\csc x+C$$

$$\int e^x dx=e^x+C$$

$$\int \cosh x dx=\sinh x+C$$

$$\int \sinh x dx=\cosh x+C$$

$$\int \text{sech}^2 x dx=\tanh x+C$$

$$\int \text{csch}^2 x dx=-\coth x+C$$

$$\int \text{sech} x\tanh x dx=-\text{sech} x+C$$

Integration of Inverse Functions

$$\int \frac{1}{\sqrt{a^2-x^2}}dx=\sin^{-1}\left(\frac{x}{a}\right)+C,\quad x^2<a^2$$

$$\int \frac{-1}{\sqrt{a^2-x^2}}dx=\cos^{-1}\left(\frac{x}{a}\right)+C,\quad x^2<a^2$$

$$\int \frac{1}{a^2+x^2}dx=\frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{-1}{a^2+x^2}dx=\cot^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{|x|\sqrt{x^2-a^2}}dx=\frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right)+C,\quad x^2>a^2$$

$$\int \frac{-1}{|x|\sqrt{x^2-a^2}}dx=\frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right)+C,\quad x^2>a^2$$

$$\int \frac{1}{\sqrt{x^2+a^2}}dx=\sinh^{-1}\left(\frac{x}{a}\right)+C$$

$$\int \frac{1}{\sqrt{x^2-a^2}}dx=\cosh^{-1}\left(\frac{x}{a}\right)+C,\quad x>a>0$$

$$\int \frac{-1}{|x|\sqrt{a^2-x^2}}dx=\frac{1}{a}\text{sech}^{-1}\left|\frac{x}{a}\right|+C,\quad 0<x<a$$

$$\int \frac{-1}{|x|\sqrt{a^2+x^2}}dx=\frac{1}{a}\text{csch}^{-1}\left|\frac{x}{a}\right|+C,\quad x\neq 0$$

$$\int \frac{1}{a^2-x^2}dx=\begin{cases}\frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right)+C, & x^2<a^2\\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right)+C, & x^2>a^2\end{cases}$$