Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ADVANCED SEMICONDUCTOR DEVICES
COURSE CODE	:	BEJ 43303
PROGRAMME CODE	:	BEJ
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONNDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **TEN** (**10**) PAGES

Q1 (a) Given a silicon *npn* BJT with these parameters:

$$N_B = 10^{15}\ cm^{-3},\ N_E = 10^{17}\ cm^{-3},\ D_E = 15\ cm/s,\ D_B = 20\ cm/s,\ T = 300\ K$$

Determine the range of $\frac{x_B}{x_E}$ ratio so that the emitter injection efficiency (γ) would be within the range of $0.95 < \gamma < 0.98$.

(7 marks)

(b) Analyse **ONE (1)** effect on emitter injection efficiency (γ) in BJT when electron flow to base region increases significantly.

(5 marks)

(c) Calculate the density of 2D electron gas (n_s) of a silicon HEMT with parameters as follows:

$$V_G = 1.2\ V,\ N_d = 10^{17}\ cm^{-3},\ d_1 = 200\ Å,\ d_0 = 50\ Å,\ \Delta d = 80\ Å,$$
$$\phi_{Bn} = 0.65\ V,\ \Delta E_C = 7.2 \times 10^{-20}\ J,\ \varepsilon_{r(AlGaAs)} = 12.3,\ T = 300\ K$$

(7 marks)

(d) Using the same parameters given in **Q1(c)**, calculate the saturated drain voltage (V_{Dsat}) and saturated drain current (I_{Dsat}) of the transistor given $\frac{Z}{L} = 1.5$.

(6 marks)

Q2 (a) Given the parameter of a silicon *n*-type DG MOSFET operating in saturation region at $T = 300\ K$ is as follows:

$$t_{ox} = 400\ Å,\ t_{Si} = 700\ Å,\ V_D = 0.2\ V,\ V_G = 0.6\ V,\ V_0 = 0.2\ V,$$
$$C_{ox} = 7 \times 10^{-8}\ F/cm,\ \frac{W}{L} = 2,\ \varepsilon_{r(ox)} = 3.9$$

Calculate both the drain current (I_D) and transconductance (g_m) of this transistor.

(7 marks)

(b) Analyse **ONE (1)** possible advantage of fin structure that contributes to improved performance in FinFET compared to planar MOSFET.

(5 marks)

(c) Consider a silicon BJT in emitter bandgap narrowing condition has the following parameters:

$$N_B = 10^{16}\ cm^{-3},\ N_E = 5 \times 10^{17}\ cm^{-3},\ D_E = 16\ cm/s,\ D_B = 25\ cm/s,$$
$$x_E = 4x_B = 2\ \mu m,\ L_E = 1.3 \times 10^{-3}\ cm,\ L_B = 1.6 \times 10^{-3}\ cm,\ T = 300\ K$$

Determine the emitter injection efficiency (γ) for this device.

(7 marks)

(d) Suppose a silicon JFET operates in channel length effect has parameters as follows:

$$V_D = 1.25\ V,\ V_{Dsat} = 1.2\ V,\ N_d = 10^{16}\ cm^{-3},\ W = 5L = 0.5\ \mu m,\ a = 1\ \mu m$$

Determine the pinch-off current (I_P') of this transistor.

(6 marks)

Q3 (a) Consider a silicon varactor with the following parameters:

$$W_D = 0.25\ \mu m,\ N_a = 10^{16}\ cm^{-3},\ N_d = 10^{16}\ cm^{-3},\ B = 10^{16}\ cm^{-3},\ m = 0$$

Calculate the reverse voltage (V_R) of this device.

(6 marks)

(b) Based on the graph in **Figure Q3(b)**, determine the breakdown voltage (V_B) and depletion width (W_D) of a gallium arsenide one-sided IMPATT diode that has electric field $E_m = 4 \times 10^5\ V/cm$.

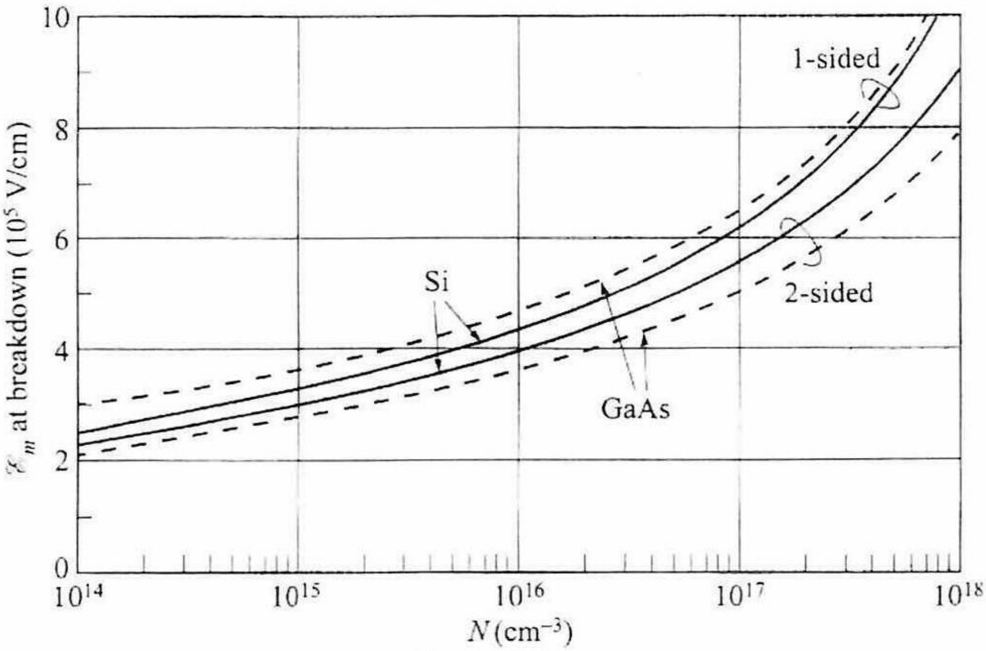

Figure Q3(b)

(7 marks)

(c) Calculate the voltage breakdown (V_B) and power density limit (P_m) of a germanium hi-lo diode that has the following parameters:

$$N_2 = 10^{14}\ cm^{-3},\ W_D = 5b = 8\ \mu m,\ E_m = 2 \times 10^5\ V/cm,\ v_s = 10^4\ cm/s$$

(7 marks)

(d) Analyse **TWO (2)** possible effect of drift region on power density in IMPATT diodes.

(5 marks)

Q4 (a) Consider a silicon thyristor is in reverse blocking mode with the following parameters:

$$N_{n1} = 10^{15}\ cm^{-3},\ W_{n1} = 12\ \mu m$$

Calculate the breakdown voltage when the depletion area covers half the n_1 region and when the depletion area covers the whole n_1 region.

(6 marks)

(b) Consider a silicon thyristor is in forward conduction mode with the following parameters:

$$N_{n1} = 6 \times 10^{15}\ cm^{-3},\ D_n = 20\ cm/s,\ D_p = 24\ cm/s,$$
$$W_{n1} = 12\ \mu m,\ W_{p2} = 4\ \mu m,\ V_{AK} = 0.8\ V$$

Calculate the recombination current density (J_{re}) of this thyristor.

(7 marks)

(c) A silicon power MOSFET at $T = 300\ K$ has the following parameters:

$$N_a = 3 \times 10^{13}\ cm^{-3},\ N_d = 10^{15}\ cm^{-3},\ V_D = 0.3\ V,\ V_G = 0\ V,\ V_{th} = 0.2\ V,$$
$$L = 0.5\ \mu m\ ,\ Z = 0.8\ \mu m,\ K_0 = 0.1,\ C_{ox} = 15\ nF$$

Determine the saturated drain current (I_{Dsat}) of this transistor.

(7 marks)

(d) Analyse the feature in U-MOSFET structure that could improve carrier flow in its channel.

(5 marks)

- END OF QUESTIONS -

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Table 1
Physical constants

Boltzmann's constant	$k = 1.38 \times 10^{-23}$ J/K $= 8.62 \times 10^{-5}$ eV/K
Electronic charge (magnitude)	$q = 1.6 \times 10^{-19}$ C
Free electron rest mass	$m_0 = 9.11 \times 10^{-31}$ kg
Permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H/m
Permittivity of free space	$\varepsilon_0 = 8.85 \times 10^{-14}$ F/cm $= 8.85 \times 10^{-12}$ F/m
Planck's constant	$h = 6.625 \times 10^{-34}$ J-s $= 4.135 \times 10^{-15}$ eV-s
Modified Planck's constant	$\hbar = 1.054 \times 10^{-34}$ J-s
Proton rest mass	$M = 1.67 \times 10^{-27}$ kg
Speed of light in vacuum	$c = 2.98 \times 10^{10}$ cm/s
Thermal voltage (T = 300 K)	$V_t = kT/q = 0.0259$ V

Table 2
Work function of selected metals

Metal	**Work function (V)**
Silver (Ag)	4.26
Aluminum (Al)	4.28
Gold (Au)	5.10
Titanium (Ti)	4.33
Tungsten (W)	4.55

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Table 3
Silicon, Gallium Arsenide and Germanium properties (T = 300 K)

Property	**Si**	**GaAs**	**Ge**
Atoms (cm^{-3})	5.0×10^{22}	4.42×10^{22}	4.42×10^{22}
Atomic weight	28.09	144.63	72.60
Density (g/cm^{-3})	2.33	5.32	5.33
Lattice constant (Å)	5.43	5.65	5.65
Melting point (°C)	1415	1238	937
Dielectric constant	11.7	13.1	16.0
Bandgap energy (eV)	1.12	1.42	0.66
Electron affinity, χ (volts)	4.01	4.07	4.13
Effective density of states in conduction band, N_c (cm^{-3})	$2.8\ 10^{19}$	4.7×10^{17}	1.04×10^{19}
Effective density of states in valence band. N_v (cm^{-3})	1.04×10^{19}	7.0×10^{18}	6.0×10^{18}
Intrinsic carrier concentration (cm^{-3})	1.5×10^{10}	1.8×10^{6}	2.4×10^{13}
Mobility (cm^2/V-s)			
Electron, μ_n	1350	8500	3900
Hole, μ_p	480	400	1900
Effective mass (density of states)			
Electrons $\left(\frac{m_n^*}{m_0}\right)$	1.08	0.067	0.55
Holes $\left(\frac{m_p^*}{m_0}\right)$	0.56	0.48	0.37

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Table 4
List of formula

BJT

1.	$i_C = i_{E1} = \frac{eD_nA_{BE}}{x_B} \cdot n_{B0} exp\left(\frac{qv_{BE}}{kT}\right)$	2.	$i_{E2} = \frac{qD_pA_{BE}}{x_E} \cdot p_{E0} exp\left(\frac{qv_{BE}}{kT}\right)$
3.	$i_E = i_{E1} + i_{E2} = I_{SE} exp\left(\frac{qv_{BE}}{kT}\right)$	4.	$\frac{i_C}{i_E} = \alpha$
5.	$\delta n_B(x) \approx \frac{n_{B0}}{x_B}\left\{\left[exp\left(\frac{qV_{BE}}{kT}\right) - 1\right](x_B - x) - x\right\}$	6.	$\delta p_E(x') \approx \frac{p_{E0}}{x_E}\left[exp\left(\frac{qV_{BE}}{kT}\right) - 1\right](x_E - x')$
7.	$\delta p_C(x'') = -p_{C0} \cdot exp\left(-\frac{x''}{L_C}\right)$	8.	$n_B(x) = \delta n_B(x) + n_{B0}$
9.	$p_E(x') = \delta p_E(x') + p_{E0}$	10.	$p_C(x'') = \delta p_C(x'') + p_{C0}$
11.	$\alpha_0 = \frac{J_C}{J_E} = \frac{J_{nC} + J_G + J_{pc0}}{J_{nE} + J_R + J_{pE}}$	12.	$\alpha = \frac{\partial J_C}{\partial J_E} = \frac{J_{nC}}{J_{nE} + J_R + J_{pE}}$
13.	$\alpha = \left(\frac{J_{nE}}{J_{nE} + J_{pE}}\right)\left(\frac{J_{nC}}{J_{nE}}\right)\left(\frac{J_{nE} + J_{pE}}{J_{nE} + J_R + J_{pE}}\right) = \gamma\alpha_T\delta$	14.	$\gamma \approx \frac{1}{1 + \frac{N_B}{N_E} \cdot \frac{D_E}{D_B} \cdot \frac{x_B}{x_E}}$ $(x_B \ll L_B,\ x_E \ll L_E)$
15.	$\alpha_T \approx \frac{1}{1 + \frac{1}{2}\left(\frac{x_B}{L_B}\right)^2}$ $(x_B \ll L_B)$	16.	$\delta = \frac{1}{1 + \frac{J_{r0}}{J_{s0}} exp\left(-\frac{qV_{BE}}{2kT}\right)}$
17.	$n_p(0) = n_{p0} exp\left(\frac{qV_{BE}}{kT}\right)$	18.	$p_p(0) = p_{p0} = N_A$
19.	$p_p(0)n_p(0) = p_{p0}n_{p0} exp\left(\frac{qV_{BE}}{kT}\right)$	20.	$n'_p(0) \approx n_{p0} exp\left(\frac{qV_{BE}}{2kT}\right)$

JFET

21.	$h = \sqrt{\frac{2\varepsilon_s(V_{bi} + V_{DS} - V_{GS})}{qN_d}}$	22.	$V_{bi} = \frac{kT}{q} ln\left(\frac{N_aN_d}{n_i^2}\right)$
23.	$V_p = \frac{qN_Da^2}{2\varepsilon_s}$	24.	$I_p = \frac{\mu_n(qN_d)^2Wa^3}{6\varepsilon_sL}$
25.	$I_D = I_P\left[3\left(\frac{V_D}{V_p}\right) - 2\left(\frac{V_D + V_{bi} + V_G}{V_p}\right)^{3/2} + 2\left(\frac{V_{bi} - V_G}{V_p}\right)^{3/2}\right]$	26.	$I_{Dsat} = I_P\left[1 - 3\left(\frac{V_{bi} - V_G}{V_p}\right)\left(1 - \frac{2}{3}\sqrt{\frac{V_{bi} - V_G}{V_p}}\right)\right]$
27.	$V_{Dsat} = V_p - (V_{bi} - V_G)$	28.	
29.	$g_m = \frac{3I_P}{V_p}\sqrt{\frac{V_{bi} - V_G}{V_p}}\left[\sqrt{\frac{V_D}{V_{bi} - V_G} + 1} - 1\right]$	30.	$g_{msat} = \frac{3I_P}{V_p}\left[1 - \sqrt{\frac{V_{bi} - V_G}{V_p}}\right]$

MESFET

31.	$R = \frac{L}{q\mu_nN_dA}$	32.	$V_{bi} = \phi_{Bn} - \frac{kT}{q} ln\left(\frac{N_C}{N_d}\right)$

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Table 4
List of formula (Cont..)

MESFET

33.	$V_{Dsat} = \frac{qN_d a^2}{2\varepsilon_s} - V_{bi} - V_G$	34.	$I_D = I_P\left[\frac{V_D}{V_P} - \frac{2}{3}\left(\frac{V_D + V_G + V_{bi}}{V_P}\right)^{3/2} + \frac{2}{3}\left(\frac{V_G + V_{bi}}{V_P}\right)^{3/2}\right]$
35.	$I_P = \frac{Z\mu_n q^2 N_d^2 a^3}{2\varepsilon_s L}$	36.	$V_P = \frac{qN_d a^2}{2\varepsilon_s}$
37.	$I_{Dsat} = I_P\left[\frac{1}{3} - \left(\frac{V_G + V_{bi}}{V_P}\right) + \frac{2}{3}\left(\frac{V_G + V_{bi}}{V_P}\right)^{3/2}\right]$	38.	$V_{Dsat} = V_P - V_G - V_{bi}$
39.	$g_m = \left.\frac{\partial I_D}{\partial V_G}\right\|_{V_D} = \frac{I_P V_D}{2V_P^2}\sqrt{\frac{V_P}{V_G + V_{bi}}}$ (*linear*)	40.	$g_m = \frac{I_P}{V_P}\left(1 - \sqrt{\frac{V_G + V_{bi}}{V_P}}\right)$ (*saturation*)

MODFET

41.	$V_P = \frac{qN_d d_1^2}{2\varepsilon_s}$	42.	$V_T = \phi_{Bn} - \frac{\Delta E_C}{q} - V_P$
43.	$n_s = \frac{\varepsilon_{AlGaAs}(V_G - V_T)}{q(d_1 + d_0 + \Delta d)}$	44.	$I = \frac{Z}{L}\mu_n C_i V_D (V_G - V_T)$
45.	$V_{Dsat} = V_G - V_T$	46.	$I_{Dsat} = \frac{Z\mu_n \varepsilon_s (V_G - V_T)^2}{2L(d_1 + d_0 + \Delta d)}$

DG MOSFET

47.	$\phi_S = \left(\frac{kT}{q}\right) ln\left(\frac{N_a N_d}{n_i^2}\right)$	48.	$I_D = 2\mu_n C_{ox}\frac{W}{L}\left(V_G - V_0 - \frac{V_D}{2}\right)V_D$ (*linear*)
49.	$I_D = 2\mu_n C_{ox}\frac{W}{L}\left\{(V_G - V_0)^2 - \frac{8rk^2T^2}{q^2}e^{q(V_G - V_0 - V_D)/kT}\right\}$ (*saturation*)		
50.	$I_D = \mu_n \frac{W}{L} kT n_i t_{Si} e^{(qV_G/kT)}\left[1 - e^{(-qV_D/kT)}\right]$ (*subthreshold*)		
51.	$r = \frac{\varepsilon_{Si} t_{ox}}{\varepsilon_{ox} t_{Si}}$	52.	$g_m = \mu_n \frac{8W\varepsilon_{Si} kT}{Lqt_{Si}}$

Varactor

53.	$N = Bx^m$	54.	$s = \frac{1}{m+2}$
55.	$W_D = \left[\frac{\varepsilon_s (m+2)(V_R + \psi_{bi})}{qB}\right]^{\frac{1}{m+2}}$	56.	$C_D = \frac{\varepsilon_s}{W_D}\left[\frac{qB\varepsilon_s^{m+1}}{(m+2)(V_R + \psi_{bi})}\right]^{\frac{1}{m+2}}$

p-i-n Diode

57.	$\tau = \frac{W}{v_s}$	58.	$C = \frac{\varepsilon_s}{W}$
59.	$V_{BD} = \mathbf{E}_m W$	60.	$J_{re} = \frac{qWn_i}{2\tau} exp\left(\frac{qV_F}{2kT}\right)$
61.	$R_{RF} = \frac{W}{q\Delta n(\mu_n + \mu_p)A} = \frac{W^2}{J_F \tau(\mu_n + \mu_p)A}$	62.	$J_F = \frac{qW\Delta n}{\tau}$

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Table 4
List of formula (Cont..)

IMPATT Diode

63.	$V_B = \frac{1}{2}\mathbf{E}_m W_D = \frac{\varepsilon_s \mathbf{E}_m^2}{2qN}$ $(1-sided)$	64.	$V_B = \frac{1}{2}\mathbf{E}_m W_D = \frac{\varepsilon_s \mathbf{E}_m^2}{qN}$ $(2-sided)$
65.	$V_B = \mathbf{E}_m W_D = \frac{qN_1 b}{\varepsilon_s}\left(W_D - \frac{b}{2}\right)$ $(Read)$	66.	$V_B = \frac{\mathbf{E}_m b}{2} + \frac{qN_2 W_D (W_D - b)}{2\varepsilon_s}$ $(Hi-Lo)$
67.	$V_B = \mathbf{E}_m b + \left(\mathbf{E}_m - \frac{qQ}{\varepsilon_s}\right)(W_D - b)$ $(Lo-Hi-Lo)$	68.	$V_m = \mathbf{E}_m W_D$
69.	$J_m = \frac{\mathbf{E}_m \varepsilon_s v_s}{W_D}$	70.	$P_m = \mathbf{E}_m^2 \varepsilon_s v_s$

Thyristors

71.	$V_B \approx 6 \times 10^{13}(N_{n1})^{-0.75}$	72.	$V_{PT} = \frac{qN_{n1}W_{n1}^2}{2\varepsilon_s}$
73.	$V_{BR} = V_B\,(1-\alpha_1)^{\frac{1}{n}}$	74.	$\alpha_1 = sech\left(\frac{W}{L_{n1}}\right)$
75.	$W = W_{n1}\left(1 - \sqrt{\frac{V_{AK}}{V_{PT}}}\right)$	76.	$I_{C1} = \alpha I_E + I_{CO1}$
77.	$I_{B1} = (1-\alpha_1)I_A - I_{CO1}$	78.	$I_{C2} = \alpha_2 I_K + I_{CO2}$
79.	$I_A = \frac{\alpha_2 I_g + I_{CO1} + I_{CO2}}{1-(\alpha_1+\alpha_2)}$	80.	$V_{BF} = V_B(1-\alpha_1-\alpha_2)^{1/n}$
81.	$V_{AK} = V_1 - \lvert V_2 \rvert + V_3$	82.	$J = \frac{qnW_i}{\tau_{eff}}$
83.	$\tau_{eff} = \frac{1}{\left(2A_r n_i^2 + \frac{1}{\tau_{p0}+\tau_{n0}}\right)}$	84.	$V_{int} = \frac{W_{n1}+W_{p2}}{(\mu_n+\mu_p)\tau_{eff}}$
85.	$J = \frac{4qn_i D_a F_L}{W_{n1}+W_{p2}} e^{\frac{V_{AK}}{2kT}}$	86.	$J_{re} = \frac{qn_i D_a}{2(W_{n1}+W_{p2})} e^{\frac{qV_{AK}}{2kt}}$
87.	$F_L = \frac{W_{n1}+W_{p2}}{\sqrt{D_a(\tau_p+\tau_n)}}$	88.	$D_a = \frac{2D_n D_p}{D_n + D_p}$

Power MOSFET

89.	$R_{on} = \frac{4V_B^2}{\varepsilon_s \mu_n E^3}$	90.	$t = \sqrt{\frac{2\varepsilon_s V_B}{qN_d}}$
91.	$I_D = \frac{Z\mu_n C_{ox}[(V_G - V_{Th})V_D]}{L}$	92.	$I_{Dsat} = \frac{Z\mu_n C_{ox}(V_G - V_{Th})^2}{L - \Delta L}$
93.	$\Delta L = K_0\sqrt{\frac{2\varepsilon_s V_D}{q}\left[\frac{N_d}{N_a(N_a+N_d)}\right]}$	94.	$g_m = \frac{\Delta I_D}{\Delta V_G} = \frac{2\mu_n C_{ox}(V_G - V_{Th})}{LW_{cell}}$

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Table 4
List of formula (Cont..)

Power MOSFET

95. $$W_{cell} = W_G + W_{poly}$$

96. $$R_0 = \frac{W_{cell} b\sqrt{V_D}}{\mu_n C_{ox}(V_G - V_{Th})^2}\left(\frac{L}{b} - V_D\right)^2$$

97. $$R_{ch} = \frac{LW_{cell}}{2\mu_n C_{ox}(V_G - V_{Th})}$$

98. $$b = K_0\sqrt{\frac{2\varepsilon_s}{q}\left[\frac{N_d}{N_a(N_a + N_d)}\right]}$$

99. $$C_{ox} = \frac{\varepsilon_{ox}}{t_{ox}}$$

100. $$C_{IN} = \frac{2x_{PL}\varepsilon_{ox}}{W_{cell}t_{ox}} + \frac{W_G\varepsilon_{ox}}{W_{cell}t_{Eox}}$$

BJT (Nonideal Effects)

101. $$n_{iE} = n_i\sqrt{e^{\left(\frac{\Delta E_g}{kT}\right)}}$$

102. $$\gamma = \frac{1}{1 + \frac{p_{E0}D_E L_B}{n_{B0}D_B L_E} \times \frac{\tanh\left(\frac{x_B}{L_B}\right)}{\tanh\left(\frac{x_E}{L_E}\right)}}$$

103. $$p_{E0} = \frac{n_{iE}^2}{N_E} = \frac{n_i^2}{N_E}e^{\left(\frac{\Delta E_g}{kT}\right)}$$

104. $$x_{dB} = W_B$$

105. $$W_B = \sqrt{\frac{2\varepsilon_s(V_{bi} + V_{pt})}{q} \cdot \frac{N_C}{N_B} \cdot \frac{1}{N_C + N_B}}$$

106. $$V_{pt} = \frac{eW_B^2}{2\varepsilon_s} \cdot \frac{N_B(N_B + N_C)}{N_C}$$

JFET (Nonideal Effects)

107. $$I_P' = \frac{\mu_n(qN_d)^2 W a^3}{6\varepsilon_s\left(L - \frac{\Delta L}{2}\right)}$$

108. $$\Delta L = \sqrt{\frac{2\varepsilon_s(V_D - V_{Dsat})}{qN_d}}$$

109. $$I_D' = I_D\left(\frac{I_P'}{I_P}\right) = I_D\left(\frac{L}{L - \frac{\Delta L}{2}}\right)$$

110. $$r_d = \frac{\partial V_D}{\partial I_D'} = \frac{\Delta V_D}{\Delta I_D'}$$

111. $$I_{Dsat}' = qN_d v_{sat}(a - h_{sat})W$$