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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	DIGITAL SIGNAL PROCESSING
COURSE CODE	:	BEJ 30603 / BEV 30603 / BEB 30503
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTION	: :	1. ANSWER ALL QUESTIONS 2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

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Q1 (a) Given a discrete signal as:

$$x[n] = r[n+3] - 2u[n] - r[n-1] + \delta[n-2] - 2u[n-2] - \delta[n-4]$$

(i) Determine the numeric sequence of $x[n]$

(6 marks)

(ii) Calculate the signal energy of $x[n]$

(2 marks)

(b) The signal $y[n] = \{\, 1, 2, \overset{\Downarrow}{1}, 2, 1, 0, -1 \,\}$ can be expressed as:

$$y[n] = 4tri\left(\frac{n-1}{4}\right) - 2rect\left(\frac{n-A}{2B}\right)$$

Propose the value of A and B, and prove your answer.

(7 marks)

(c) Calculate the periodic convolution of $x_p[n] = \{\overset{\Downarrow}{3}, -5, 6, 2\,\}$ and $h_p[n] = \{\overset{\Downarrow}{-7}, 1\,, -4, 8\,\}$ using the cyclic method.

(5 marks)

Q2 (a) The sampling operation leads to a potential loss of information in a digital signal processing implementation.

(i) Explain briefly how the information loss can be avoided.

(2 marks)

(ii) Describe the aliasing in the sampling process.

(5 marks)

(b) Describe the function of quantizer in an analog to digital converter (ADC) system.

(2 marks)

(c) You are given the following analog signal:

$$x(t) = \frac{5}{1.4(t+0.55)} - 2$$

Analyse the process of ADC for the signal $x(t)$ to estimate the quantization signal-to-noise ratio (SNR_Q) in dB for the first 6 samples (the first sample is at t = 0 s) using rounding method. The dynamic range is 5 V with 3 bits length and the sampling interval is 0.2 second. Use the sampling value with **FOUR (4)** decimal points.

(11 marks)

Q3 (a) Define the N-point Discrete Fourier Transform (DFT) and the inverse DFT.

(4 marks)

(b) Let $Y_{DFT}[n] = \{\overset{\Downarrow}{7}, j3,\ 1, -j3\ \}$. Using IDFT definition, find $y[n]$.

(5 marks)

(c) Given the DFT pair $x[n] = \{\overset{\Downarrow}{4}, 1, 3, 2\ \} \quad \Leftrightarrow X_{DFT}[k] = \{\ \overset{\Downarrow}{10}, 1 + j, 4, A\ \}$. Using DFT property,

(i) Find A.

(1 mark)

(ii) Calculate $Y_{DFT}[k]$ if $y[n] = x[n + 3]$

(5 marks)

(d) Produce a flowchart showing all twiddle factors and values at intermediate nodes to compute the discrete Fourier transform (DFT) of $x[n] = \{\overset{\Downarrow}{-2}, 5, -4, 7\ \}$ by using the 4-point decimation in time (DIT) algorithm.

(5 marks)

Q4 (a) z-Transform is an important tool in a discrete system analysis.

(i) Define the z-Transform.

(2 marks)

(ii) Describe and illustrate the Region of Convergence (ROC) for z-Transform of unit step, $u[n]$

(3 marks)

(b) The z-transform of signal $x[n] = \alpha^n u[n]$ is given as $X(z) = \frac{z}{z-\alpha}$ with the region of convergence (ROC) of $|z| > \alpha$. Find the z-transforms and specify their ROC for the following:

(i) $y[n] = x[3 + n]$

(2 marks)

(ii) $p[n] = (-0.4)^n x[n]$

(2 marks)

(c) Given $X(z) = \dfrac{z^3}{(z-0.6)(z-0.2)^2}$. Analyse $X(z)$ to determine the value of signal $x[n]$ for n = 0. Assume the signal is right-sided. Use the partial fraction expansion method in your answer.

Hints:

$$H(z) = \frac{N(z)}{(z-p_1)(z-p_2)...(z-p_n)} = \frac{A_1}{(z-p_1)} + \frac{A_2}{(z-p_2)} + .. + \frac{A_n}{(z-p_n)}$$

$$A_i = (z-p_i)H(z)\big|_{z=p_i}$$

$$\frac{A_{i1}}{(z-p_i)} + \frac{A_{i2}}{(z-p_i)^2} + .. + \frac{A_{ir}}{(z-p_i)^r}$$

$$A_{ir} = (z-p_i)^r H(z)\big|_{z=p_i},\ A_{i(r-1)} = \frac{d}{dz}(z-p_i)^r H(z)\big|_{z=p_i},\ A_{i(r-k)} = \frac{1}{k!}\frac{d^k}{dz^k}(z-p_i)^r H(z)\big|_{z=p_i}$$

(11 marks)

Q5 (a) List **FOUR(4)** advantages of the digital filter than analog filter.

(4 marks)

(b) Describe with an appropriate diagram the steps to design an Infinite Impulse Response (IIR) filter using the response matching approach.

(6 marks)

(c) Design a lowpass Finite Impulse Response (FIR) filter with cutoff frequency of 6 kHz and sampling rate of 20 kHz. By using the filter length of 7 and Bartlett window, calculate the impulse response $h_N[n]$, window sequence $w_N[n]$ and windowed impulse respose $h_W[n]$.

(7 marks)

(d) Design a highpass FIR filter based on the specification in **Q5(c).**

(3 marks)

-END OF QUESTIONS -

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Table 1: Properties of the *N*-Sample DFT

Property	**Signal**	**DFT**
Shift	$x[n-n_o]$	$X_{DFT}[k]e^{-j2\pi k n_o/N}$
Shift	$x[n-0.5N]$	$(-1)^k X_{DFT}[k]$
Modulation	$x[n]e^{j2\pi n k_o/N}$	$X_{DFT}[k-k_o]$
Modulation	$(-1)^n x[n]$	$X_{DFT}[k-0.5N]$
Folding	$x[-n]$	$X_{DFT}[-k]$
Product	$x[n]y[n]$	$\frac{1}{N}X_{DFT}[k]\otimes Y_{DFT}[k]$
Convolution	$x[n]\otimes y[n]$	$X_{DFT}[k]Y_{DFT}[k]$
Correlation	$x[n]\otimes\otimes y[n]$	$X_{DFT}[k]Y^*_{DFT}[k]$
Central Ordinates	$x[0]=\frac{1}{N}\sum_{k=0}^{N-1}X_{DFT}[k]$, $\quad X_{DFT}[0]=\sum_{n=0}^{N-1}x[n]$	
Central Ordinates	$x\left[\frac{N}{2}\right]=\frac{1}{N}\sum_{k=0}^{N-1}(-1)^k X_{DFT}[k]$ (N even), $X_{DFT}\left[\frac{N}{2}\right]=\sum_{n=0}^{N-1}(-1)^n x[n]$ (N even)	

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Table 2: Properties of the z- transform

Property	**Signal**	**z-transform**
Linearity	$a_1x_1(n) + a_2x_2(n)$	$a_1X_1(z) + a_2X_2(z)$
Time reversal	$x(-n)$	$X(z^{-1})$
Time shifting	i) $x(n-k)$ ii) $x(n+k)$	i) $z^{-k}X(z)$ ii) $z^{k}X(z)$
Convolution	$x_1(n) * x_2(n)$	$X_1(z)X_2(z)$
Correlation	$r_{x_1x_2}(l) = \sum_{n=-\infty}^{\infty} x_1(n)x_2(n-l)$	$R_{x_1x_2}(z) = X_1(z)X_2(z^{-1})$
Scaling	$a^n x(n)$	$X(a^{-1}z)$
Differentiation	$nx(n)$	$z^{-1}\frac{dX(z)}{dz^{-1}}$ or $-z\frac{dX(z)}{dz}$
Time differentiation	$x(n) - x(n-1)$	$X(z)(1 - z^{-1})$
Time integration	$\sum_{k=0}^{\infty} X(k)$	$X(z) = \left(\frac{z}{z-1}\right)$
Initial value theorem	$\lim_{n \to 0} x(n)$	$\lim_{\|z\| \to \infty} X(z)$
Final value theorem	$\lim_{n \to \infty} x(n)$	$\lim_{\|z\| \to 1} \left(\frac{z-1}{z}\right) X(z)$

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Table 3: z-Transform Pairs

Signal $x(t)$	Sequence $x(n)$	z-Transform $X(z)$
$\delta(t)$	$\delta(n)$	1
$\delta(t - k)$	$\delta(n - k)$	z^{-k}
$u(t)$	$u(n)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
	$-u(-n - 1)$	$\frac{1}{1-z^{-1}} = \frac{z}{z-1}$
$r(t) = tu(t)$	$nu(n)$	$\frac{z^{-1}}{\left(1-z^{-1}\right)^2} = \frac{z}{(z-1)^2}$
	$a^n u(n)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$-a^n u(-n - 1)$	$\frac{1}{1-az^{-1}} = \frac{z}{z-a}$
	$na^n u(n)$	$\frac{az}{(z-a)^2}$
	$-na^n u(-n - 1)$	$\frac{az}{(z-a)^2}$
e^{-at}	e^{-an}	$\frac{1}{1-e^{-a}z^{-1}} = \frac{z}{z-e^{-a}}$
t^2	$n^2 u(n)$	$z^{-1}\frac{\left(1+z^{-1}\right)}{\left(1-z^{-1}\right)^3} = \frac{z(z+1)}{(z-1)^3}$
te^{-at}	ne^{-an}	$\frac{z^{-1}e^{-a}}{\left(1-e^{-a}z^{-1}\right)^2} = \frac{ze^{-a}}{\left(z-e^{-a}\right)^2}$
$sin\omega_o t$	$sin\omega_o n$	$\frac{z\sin\omega_o}{z^2 - 2z\cos\omega_o + 1}$
$cos\omega_o t$	$cos\omega_o n$	$\frac{z(z-\cos\omega_o)}{z^2 - 2z\cos\omega_o + 1}$

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Table 4: Digital- to- digital Transformations

Form	Band Edges	Mapping $z \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-\alpha}{1-\alpha z}$	$\alpha = \frac{\sin[0.5(\Omega_D - \Omega_C)]}{\sin[0.5(\Omega_D + \Omega_C)]}$
Lowpass to highpass	Ω_C	$\frac{-(z+\alpha)}{1+\alpha z}$	$\alpha = \frac{-\cos[0.5(\Omega_D + \Omega_C)]}{\cos[0.5(\Omega_D - \Omega_C)]}$
Lowpass to bandpass	$[\Omega_1, \Omega_2]$	$\frac{-(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \frac{\tan(0.5\Omega_D)}{\tan[0.5(\Omega_2 - \Omega_1)]}$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha K}{K+1},\ A_2 = \frac{K-1}{K+1}$
Lowpass to bandstop	$[\Omega_1, \Omega_2]$	$\frac{(z^2 + A_1 z + A_2)}{A_2 z^2 + A_1 z + 1}$	$K = \tan(0.5\Omega_D)\tan[0.5(\Omega_2 - \Omega_1)]$ $\alpha = \frac{-\cos[0.5(\Omega_2 + \Omega_1)]}{\cos[0.5(\Omega_2 - \Omega_1)]}$ $A_1 = \frac{2\alpha}{K+1},\ A_2 = \frac{1-K}{1+K}$

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Table 5: Direct Analog- to- digital Transformations for Bilinear Design

Form	Band Edges	Mapping $s \rightarrow$	Parameters
Lowpass to lowpass	Ω_C	$\frac{z-1}{C(z+1)}$	$C = \tan(0.5\Omega_C)$
Lowpass to highpass	Ω_C	$\frac{C(z+1)}{z-1}$	$C = \tan(0.5\Omega_C)$
Lowpass to bandpass	$\Omega_1 < \Omega_0 < \Omega_2$	$\frac{z^2-2\beta z+1}{C(z^2-1)}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$
Lowpass to bandstop	$\Omega_1 < \Omega_0 < \Omega_2$	$\frac{C(z^2-1)}{z^2-2\beta z+1}$	$C = \tan[0.5(\Omega_2 - \Omega_1)]$, $\beta = \cos\Omega_0$ or $\beta = \frac{\cos[0.5(\Omega_2+\Omega_1)]}{\cos[0.5(\Omega_2-\Omega_1)]}$

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Table 6: Windows for FIR filter design.

Window	**Expression** $w_N[n]$, $-0.5(N-1) \leq n \leq 0.5(N-1)$
Boxcar	1
Cosine	$\cos\left(\frac{n\pi}{N-1}\right)$
Riemann	$\text{sinc}^L\left(\frac{2n}{N-1}\right), L>0$
Bartlett	$1-\frac{2\lvert n\rvert}{N-1}$
Von Hann (Hanning)	$0.5+0.5\cos\left(\frac{2n\pi}{N-1}\right)$
Hamming	$0.54+0.46\cos\left(\frac{2n\pi}{N-1}\right)$
Blackman	$0.42+0.5\cos\left(\frac{2n\pi}{N-1}\right)+0.08\cos\left(\frac{4n\pi}{N-1}\right)$
Kaiser	$\frac{I_0\left(\pi\beta\sqrt{1-4\left(\frac{n}{N-1}\right)^2}\right)}{I_0(\pi\beta)}$

Table 7: Characteristics of the windowed spectrum for various windows.

Window	**Peak Ripple** $\delta_p = \delta_s$	**Passband Attenuation** A_{WP}(dB)	**Peak Sidelobe Attenuation** A_{WS} (dB)	**Transition Width** $F_{WS} \approx C/N$
Boxcar	0.0897	1.5618	21.7	$C = 0.92$
Cosine	0.0207	0.3600	33.8	$C = 2.10$
Riemann	0.0120	0.2087	38.5	$C = 2.50$
von Hann (Hanning)	0.0063	0.1103	44.0	$C = 3.21$
Hamming	0.0022	0.0384	53.0	$C = 3.47$
Blackman	1.71×10^{-4}	2.97×10^{-3}	75.3	$C = 5.71$

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Identity

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

$$\cos\theta = \frac{1}{2}\left(e^{j\theta} + e^{-j\theta}\right)$$

$$\sin\theta = \frac{1}{j2}\left(e^{j\theta} - e^{-j\theta}\right)$$

Finite Summation Formula

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$

$$\sum_{k=0}^{n} \alpha^k = \frac{1-\alpha^{n+1}}{1-\alpha}, \quad \alpha \neq 1$$

$$\sum_{k=0}^{n} k\alpha^k = \frac{\alpha\left[1-(n+1)\alpha^n + n\alpha^{n+1}\right]}{(1-\alpha)^2}$$

$$\sum_{k=0}^{n} k^2\alpha^k = \frac{\alpha\left[(1+\alpha)-(n+1)^2\alpha^n + (2n^2+2n-1)\alpha^{n+1} - n^2\alpha^{n+2}\right]}{(1-\alpha)^3}$$

Infinite Summation Formula

$$\sum_{k=0}^{\infty} \alpha^k = \frac{1}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} \alpha^k = \frac{\alpha}{1-\alpha}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k\alpha^k = \frac{\alpha}{(1-\alpha)^2}, \qquad |\alpha| < 1$$

$$\sum_{k=1}^{\infty} k^2\alpha^k = \frac{\alpha^2+\alpha}{(1-\alpha)^3}, \qquad |\alpha| < 1$$

$$\sum_{k=-\infty}^{\infty} e^{-\alpha|k|} = \frac{1+e^{-\alpha}}{1-e^{-\alpha}}, \qquad \alpha > 0$$