UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	ALGEBRA
COURSE CODE	:	DAS 10103
PROGRAMME	:	1DAA / 1DAM / 1DAE / 1DAU / 1DAT
EXAMINATION DATE	:	DECEMBER 2014/ JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	A) ANSWER **ALL** QUESTIONS IN PART A B) ANSWER **THREE (3)** QUESTIONS IN PART B

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

PART A

Q1 (a) Given u = $-2i + 5j + 5k$ and v = $2i - 2j - k$, find $|u+v|$.

(3 marks)

(b) If a = $2i - 3j + 5k$, b = $3i - j + 4k$ and c = $5i + 2j - k$, find

(i) a•b
(ii) b×c
(iii) a•(b×c)

(8 marks)

(c) Find an equation of the line that passes through P(1, 2, 0) and Q(– 3 , 0, 1).

(3 marks)

(d) Find the vector equation of the plane containing P(4, – 1, 2), Q(2, 0, 3) and R(– 1 , 0, 2).

(6 marks)

Q2 (a) If $z = 2+3i$, express in the form $a + ib$, the complex number $\dfrac{z^2}{(3-z)^2}$.

(5 marks)

(b) Given $z_1 = 3 + 2i$ and $z_2 = 4 - 3i$. Show that $|z_1 z_2| = |z_1|\,|z_2|$.

(6 marks)

(c) Find the four roots of $z = 3 + 4i$.

(9 marks)

PART B

Q3 (a) Solve the following exponential equation: $\left(\dfrac{1}{3}\right)^{-2-2x} = 729^{-1}$

(6 marks)

(b) Simplify $\dfrac{\sqrt[5]{64x^7y^9}}{\sqrt{4x^2y^2}\,\sqrt[3]{27x^5y}}$

(6 marks)

(c) Find the value of x without using calculator $\log_2(\log_6\sqrt{x^2+x}) = -1$.

(8 marks)

Q4 (a) Find the root of the equation $e^{-x} - x = 0$ in the interval $[0,1]$ accurate to within $\varepsilon = 0.005$ using secant method. Show all your calculation in three decimal places.

(7marks)

(b) Express $\dfrac{2x+1}{(x-1)^2(2x-5)}$ in partial fraction.

(7 marks)

(c) Expand $\dfrac{1}{\sqrt{1-2x}}$ in ascending powers of x until the term involving x^3.

(6 marks)

Q5 (a) Find the sum of $\sum_{n=1}^{8}\left(\dfrac{n^2}{4}+\dfrac{2}{3}n-1\right)$.

(5 marks)

(b) Given that the second term of an arithmetic sequence is 4 and its fourth term is -2.

(i) Find the value of first term, a and its common difference, d.

(3 marks)

(ii) Hence, calculate the sum for this series.

(2 marks)

(c) A geometric sequence is defined as 1, 1.1, 1.21, 1.331,... Find

(i) the value of common ratio, r,

(3 marks)

(ii) the tenth term, T_{10} and

(2 marks)

(d) Given an infinite geometric series $7+\dfrac{14}{5}+\dfrac{28}{25}+\ldots$

(i) State whether this series converges or diverges.

(3 marks)

(ii) If it is converges, evaluate its summation, S_∞.

(2 marks)

Q6 (a) Without using calculator, find the exact value of $\cos 15°$.

(5 marks)

(b) Solve $\cos\theta = \sin\left(\theta + \frac{\pi}{3}\right)$ for $0 \le \theta \le 2\pi$.

(7 marks)

(c) Given $5\sin\theta + 12\cos\theta = r\sin(\theta + \alpha)$ and $0° \le \theta \le 360°$

(i) Find r and α

(3 marks)

(ii) Thus, find the value of θ if $6\sec\theta - 5\tan\theta = 12$.

(5 marks)

Q7 (a) Given the matrices $A = \begin{bmatrix} 1 & 7 & 3 \\ 0 & 5 & 2 \\ 3 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -3 & -4 & 5 \\ 1 & -5 & 4 \\ -1 & 13 & -11 \end{bmatrix}$ and $C = \begin{bmatrix} 5 & -7 & -1 \\ 6 & -8 & -2 \\ -15 & 21 & 51 \end{bmatrix}$.

(i) Find $A(B + C)$ and AC.

(3 marks)

(ii) Find the inverse of the matrix A.

(5 marks)

(b) Given

$$\begin{aligned} x + 2y &= 5 \\ 3x + 2y + z &= 10 \\ 2x + 4y + z &= 13 \end{aligned}$$

(i) Write the matrix equation $AX = B$ of the above system of equation.

(1 mark)

(ii) Find the determinant of matrix A.

(2 marks)

(iii) Solve the above system for x, y and z by using Gauss-Jordan elimination method. Do this following operation in order: $R_2 - 3R_1$, $R_3 - 2R_1$, $-\frac{1}{4}R_2$, $R_2 + \frac{1}{4}R_3$, $R_1 - 2R_2$.

(9 marks)

- END OF QUESTION -

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Formulae

Polynomials

$$\log_a x = \frac{\log_a x}{\log_a b}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c, \qquad x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

Sequence and Series

$$\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$u_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d], \qquad S_n = \frac{n}{2}(a + u_n)$$

$$u_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r - 1}, r > 1 \text{ or } S_n = \frac{a(1 - r^n)}{1 - r}, r < 1, \quad S_\infty = \frac{a}{1 - r}.$$

$$u_n = S_n - S_{n-1}$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \ldots\ldots$$

Trigonometry

$$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$$

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta} \qquad \tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}$$

$$\sin 2\theta = 2\sin\theta\cos\theta, \quad \cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$$

$$\sin\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{2}}, \quad \cos\frac{\theta}{2} = \pm\sqrt{\frac{1 + \cos\theta}{2}}, \quad \tan\frac{\theta}{2} = \pm\sqrt{\frac{1 - \cos\theta}{1 + \cos\theta}}$$

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$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha) = r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) = (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta$ and

$a = r\cos\alpha$ and $b = r\sin\alpha$

$$x_1^{(k+1)} = \frac{b_1 - a_{12}x_2^{(k)} - a_{13}x_3^{(k)}}{a_{11}}, \quad x_2^{(k+1)} = \frac{b_2 - a_{21}x_1^{(k+1)} - a_{23}x_3^{(k)}}{a_{22}}, \quad x_3^{(k+1)} = \frac{b_3 - a_{31}x_1^{(k+1)} - a_{32}x_2^{(k+1)}}{a_{33}}$$

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \ |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$Adj(A) = (c_{ij})^T \qquad A^{-1} = \frac{1}{|A|}Adj(A)$$

Vector

$$\cos\theta = \frac{\boldsymbol{a}\bullet\boldsymbol{b}}{|\boldsymbol{a}||\boldsymbol{b}|} \quad \text{or} \quad \boldsymbol{a}\bullet\boldsymbol{b} = x_1x_2 + y_1y_2 + z_1z_2, \ \boldsymbol{a}\times\boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ x_1 & y_1 & z_1 \\ x_2 & y_2 & z_2 \end{vmatrix}$$

$|a| = \sqrt{x^2+y^2+z^2}$, $x = x_0 + a_1 t$, $y = y_0 + a_2 t$, $z = z_0 + a_3 t$ and $\dfrac{x - x_o}{a_1} = \dfrac{y - y_o}{a_2} = \dfrac{z - z_o}{a_3}$

$A(x - x_o) + B(y - y_o) + C(z - z_o) = 0$

Complex number

$r = \sqrt{x^2+y^2}$ $\qquad \tan\theta = \dfrac{y}{x}$

If $z = re^{i\theta}$, then $z^n = r^n e^{in\theta}$

If $z = re^{i\theta}$, then $z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\left(\frac{\theta+2k\pi}{n}\right)i}$.

If $z = r(\cos\theta + i\sin\theta)$ then $z^n = r^n(\cos n\theta + i\sin n\theta)$

If $z = r(\cos\theta + i\sin\theta)$ then $z^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\dfrac{(\theta+2k\pi)}{n} + i\sin\dfrac{(\theta+2k\pi)}{n}\right)$