Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ENGINEERING MATHEMATICS
COURSE CODE	:	BFC 25103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JULY/ AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	1.	ANSWER ALL QUESTIONS
	2.	THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.**
	3.	STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) By applying the integral definition, find the Laplace transforms for each of the following:

(i) $f(t) = 12.$

(2 marks)

(ii) $f(t) = e^{8t}$

(3 marks)

(b) Determine the Inverse Laplace transforms using a partial fraction of function below:

$$F(s) = \frac{(s+2)}{(s^2+6s+8)}$$

(5 marks)

(c) Determine the value of b if the function is continuous at every x.

$$f(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$$

(5 marks)

(d) (i) Find the first and second partial derivatives of

$$f(x, y) = 4x^3 - 5xy^2 + 3y^3$$

(4 marks)

(ii) A civil engineer is designing a bridge that needs to withstand the forces of wind and traffic. The bridge deck is supported by a series of beams, which are subject to bending under load. The deflection of the beams is given by the equation $w(x, y) = 0.01x^2y^2 - 0.1xy^3 + 0.5x^2 + 0.5y^2$, where x and y are the coordinates of a point on the beam. Determine the second-order partial derivatives of $w(x, y)$ with respect to x and y.

(6 marks)

Q2 (a) By using double integrals, sketch and find the surface area of the portion of the paraboloid $z = x^2 + y^2$ and below the plane $z = 1$.

(10 marks)

(b) You are designing a silo for the grain storage, identify the volume of geometry bounded by tetrahedron which is enclosed by the coordinate planes and the plane $2x + y + z = 4$ using triple integrals.

(15 marks)

Q3 (a) Based on **Figure Q3(a)**, find the mass, moments and the center of mass of the lamina of density $\rho(x, y) = x + y$ occupying the region R under the curve $y = x^2$ in the interval $0 \ll x \ll 2$.

(10 marks)

(b) Based on the following equations, sketch the graph of two surface which are $x^2 + y^2 = 9$ and $z = y^2$. Then, determine the equation of the intersection using a vector valued function.

(10 marks)

(c) Determine the velocity, speed and acceleration of a particle given by the position function:

$$r(t) = 3\cos t\ \mathbf{i} + 2\sin t\ \mathbf{j}$$

(5 marks)

Q4 (a) Find the unit tangent vector and the principal unit normal vector at each point on the graph of the vector function

$$R(t) = e^t\boldsymbol{i} + e^{-t}\boldsymbol{j} + \sqrt{2}t\boldsymbol{k}$$

(10 marks)

(b) Show that $\boldsymbol{F} = (y^2 - z^2 + 3yz - 2x)\boldsymbol{i} + (3xz + 2xy)\boldsymbol{j} + (3xy - 2xz + 2z)\boldsymbol{k}$ is conservative. Find:

(i) Scalar potential.

(6 marks)

(i) Work done by **F** in moving a particle from $(1,0,1)$ to $(2,1,3)$.

(4 marks)

(c) Show that $\iint_S F.dS = \frac{3}{8}$, where $F = yz\boldsymbol{i} - xz\boldsymbol{j} + xy\boldsymbol{k}$ and S is the part of the sphere $x^2 + y^2 + z^2 = 1$ that lies in the first octant.

(5 marks)

– END OF QUESTIONS –

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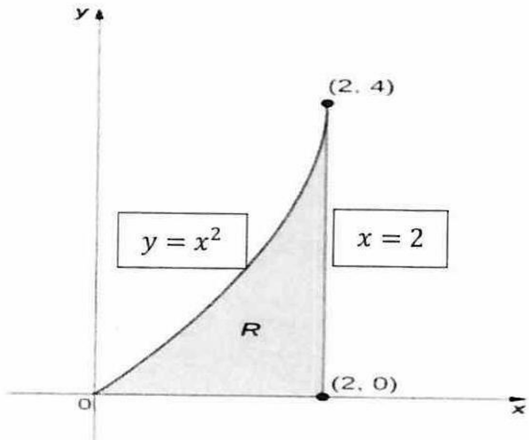

Figure Q3(a): The lamina of density $\rho(x, y) = x + y$ occupying the region R under the curve $y = x^2$

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
Ce^{ax}	$x^r(ke^{ax})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}dx$, $u_2 = \int \frac{y_1 f(x)}{W}dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^{\infty} f(t)e^{-st}\,dt = F(s)$$

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s) - y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s) - sy(0) - \dot{y}(0)$
$t^n f(t),\ n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$		

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Formulae

Tangent Plane: $z - z_0 = f_x(x_o, y_0)(x - x_0) + f_y(x_o, y_0)(y - y_0)$

Local Extreme Value: $G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - \left[f_{xy}(x,y)\right]^2$

Case	$G(a,b)$	Result
1	$G(a,b) > 0$ $f_{xx}(a,b) < 0$	$f(x,y)$ has a local maximum value at (a,b)
2	$G(a,b) > 0$ $f_{xx}(a,b) > 0$	$f(x,y)$ has a local minimum value at (a,b)
3	$G(a,b) < 0$	$f(x,y) has\ a\ saddle\ point\ at\ (a,b)$
4	$G(a,b) = 0$	inconclusive

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = \tan^{-1}(\frac{y}{x})$ and
$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical coordinate: $x = r\cos\theta$, , $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)d\ dz\ dr\ d\theta$

Spherical coordinate: $x = \rho \sin\sin\phi \cos\cos\theta$, $y = \rho \sin\sin\phi \sin\sin\theta$, $z = \rho \cos\cos\theta$, $x^2 + y^2 + z^2 = \rho^2$, $0 \ll \theta \ll 2\pi$, $0 \ll \phi \ll \pi$ and $\iiint f(x,y,z)dV = \iiint f(\rho,\phi,\theta)\rho^2 \sin\sin\phi\ d\rho d\phi d\theta$

For lamina

Mass, $m = \iint_R \delta(x,y)\ dA$

Moment of mass: y-axis: $M_y = \iint_R x\delta(x,y)\ dA$ x-axis, $M_x = \iint_R y\delta(x,y)\ dA$

Center of mass, $\left(\underline{x}, \underline{y}\right) = (\frac{M_y}{m}, \frac{M_x}{m})$

Centroid for homogenous lamina: $\underline{x} = \frac{1}{area}\iint_R x\ dA$ $\underline{y} = \frac{1}{area}\iint_R y\ dA$

Moment inertia:

Y-axis: $I_y = \iint_R x^2\delta(x,y)\ dA$ x-axis: $I_x = \iint_R y^2\delta(x,y)\ dA$

Z-axis (or origin): $I_z = I_0 = \iint_R (x^2 + y^2)\delta(x,y)\ dA = I_x + I_y$

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For solid

Mass, $m = \iiint_G \delta(x,y)\, dV$

Moment of mass:

yz-plane: $M_{yz} = \iiint_G x\, \delta(x,y,z)\, dV$

xz-plane: $M_{xz} = \iiint_G y\, \delta(x,y,z)\, dV$

xy-plane: $M_{xy} = \iiint_G z\, \delta(x,y,z)\, dV$

Center of gravity, $\left(\underline{x},\ \underline{y}, \underline{z}\right) = (\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m})$

Moment inertia:

$$I_y = \iiint_G (x^2 + z^2)\, \delta(x,y,z) dV$$
$$I_x = \iiint_G (y^2 + z^2)\, \delta(x,y,z) dV$$
$$I_z = \iiint_G (x^2 + y^2)\, \delta(x,y,z) dV$$

Directional derivative: $D_u f(x,y) = \left(f_x i + f_y j\right) \cdot u$

Let $F(x,y,z) = Mi + Nj + Pk$ is vector field, then the divergence of $F = \nabla \cdot F = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The curl of

$$F = \nabla \times F = \left| i\ j\ k\ \frac{\partial}{\partial x}\ \frac{\partial}{\partial y}\ \frac{\partial}{\partial z}\ M\ N\ P \right| = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right) i - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right) j + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right) k$$

Let C is a smooth curve given by $r(t) = x(t)i + y(t)j + z(t)k, t$ is parameter, then

The unit tangent vector; $T(t) = \frac{r'(t)}{\|r'(t)\|}$

The unit normal vector: $N(t) = \frac{T'(t)}{\|T'(t)\|}$

The binormal vector: $B(t) = T(t) \times N(t)$

The curvature: $K = \frac{T'(t)}{\|r'(t)\|} = \frac{\|r' \times r''(t)\|}{\|r'(t)\|^3}$

The radius of curvature: $\rho = 1/K$

Green Theorem: $\oint_C \quad M(x,y)\, dx + N(x,y)\, dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} dA$

Gauss Theorem: $\iint_\sigma F \cdot n\, dS = \iiint_G \nabla . F\, dV$

Stokes Theorem: $\oint_C \quad F. dr = \iint_\sigma (\nabla \times F) \cdot n\, dS$

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Arc length, If $r(t) = x(t)i + y(t)j,\ t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \|r'(t)\|dt = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\,dt$$

If $r(t) = x(t)i + y(t)j + z(t)k,\ t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2}\,dt$$

Trigonometric and Hyperbolic Identities

Trigonometric	**Hiperbolic**
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \mathrm{cosec}^2 x$	$cosh^2\, x - sinh^2\, x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - tanh^2\, x = sech^2\, x$
$\cos 2x = \cos^2 x - \sin^2 x$	$cosh^2\, x - 1 = csch^2\, x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = cosh^2\, x + sinh^2\, x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2cosh^2\, x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2sinh^2\, x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$