Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	: NUMERICAL METHODS
COURSE CODE	: BFC 25203
PROGRAMME CODE	: BFF
EXAMINATION DATE	: JULY/AUGUST 2023
DURATION	: 3 HOURS
INSTRUCTION	: 1.ANSWER ALL QUESTIONS 2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) **Table Q1 (a)** presents the rainfall data at Pasir Gudang, Johor over period in December. Do all calculations rounded to 2 decimal places.

(i) Determine the rainfall data in 2018 using the Newton's Divided Difference Method.

(12 marks)

(ii) Based on **Q1(a)(i)** verify that the missing data in 2021 and provide a conclusion.

(2 marks)

(b) As a design engineer, you have been assigned to estimate evaporation rates based on the required amount of water for irrigation purposes. The slope of the saturation vapor pressure curve (e_s) at the air temperature (T) is the formula used to make this estimation. The data in **Table Q1(b)** have been collected to assist in this design estimation, and all calculations should be rounded to 3 decimal places.

(i) Choose the appropriate formula to determine the approximation of slope of the saturation vapor at $T = 22°\text{C}$ using selected 3-point central and 5-point central formulas. Justify the derivative formula to support your answer.

(6 marks)

(ii) Solve the exact solution of the evaporation rates is $0.0625\,x^3 - 1.125x^2 - \sin(x) - 119.81$.

(2 marks)

(iii) Based on answer in **Q1(b)(i)** and **Q1(b)(ii)**, justify the method that could be capable of generating the most accurate approximation.

(3 marks)

Q2 (a) Given the equation of the irregular curve of stream, $y = 9x^2 \sin(x)$. Approximate the stream cross-sectional area of irregular shapes from $x = 0$ to $x = \frac{\pi}{2}$ into 7 equal intervals by using accurate Simpson's rule and express the absolute error. Do all calculations in 3 decimal places.

(13 marks)

(b) Given $\int_a^b f(t)\,dt = \frac{b-a}{2}\int_{-1}^{1} g(x)dx$

(i) By taking $t = \frac{(b-a)x+(b+a)}{2}$, show that the two integrals above are equivalent with the limit of integral from *a* to *b* to *-1* to *1*.

(9 marks)

(ii) The velocity of a car at time t minutes is given by $-\frac{\sin t}{t+1}$, determine the displacement traveled from t=0 to t=1 by using the 2-point and 3-point Gauss Quadrature formula.

(6 marks)

Q3 The stability of the bridge construction can be calculated and determined by the natural frequency of a bridge system (dominant eigenvalue) and its corresponding eigenvector in the matrix form as:

$$C = \begin{bmatrix} 5 & -2 & 1 \\ -2 & 2 & 0 \\ 2 & -2 & 3 \end{bmatrix}$$

Use $v^{(0)} = (1 \quad 0 \quad 1)^T$ and stop the iteration until $|m_{k+1} - m_k| < 0.0005$. Do all calculations in 3 decimal places.

(13 marks)

Q4 (a) Noise pollution in Kuala Lumpur has become increasingly serious due to the development of transportation systems, construction and industrial activities. Malaysia's Department of Environment (DOE) had imposed a noise limit of 60 decibels (dB) to protect the population from this urban noise. The noise level in Kuala Lumpur is given by the following differential equation:

$$\frac{dL}{dt} = \frac{1}{0.008 \cdot \ln(10) \cdot L}$$

where L is sound pressure level in decibel (dB)

(i) Estimate the noise level in Kuala Lumpur from year 2022 to 2030 by using fourth-order Runge Kutta Method with increment of 2 years. Assume that the noise level in 2022 is 50.0 dB. Do all calculations in 3 decimal places.

(12 marks)

(ii) Based on the answer in **Q4 (a)**, are the noise levels acceptable in future built environment of Kuala Lumpur? Justify your answer.

(2 marks)

(b) The steady state of the temperature of a 4 meters steel rod AB in **Figure Q4(b)**, with taking $\Delta x = h = 1$, satisfies the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial u^2}{\partial x^2}$$

with the boundary condition,

$$u(0,t) = 75, u(4,t) = 50, 0 < t < 0.6$$

and initial condition

$$u(x,0) = 0, 0 < x < 4$$

At t= 0.6s, the left end of point A is suddenly rose to 87.5°C while the right end points are kept at the temperature at 75°C. By taking $k = \Delta t = 0.2$s until $t = 0.6$s only, use the Crank-Nicolson method to solve the heat equation.

(20 marks)

-END OF QUESTIONS-

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Table Q1(a)

Year	2014	2015	2016	2017	2019	2020
Precipitation (mm)	11.88	49.70	76.59	40.52	366.16	238.96

Table Q1(b)

Air Temperature, T (°C)	e_s (mm Hg)
20	17.53
21	18.65
22	19.82
23	21.05
24	22.37
25	23.75

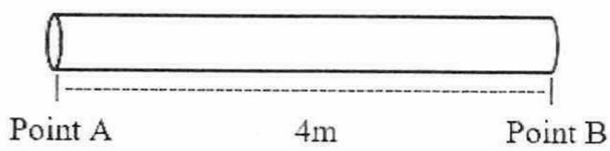

Figure Q4(b)

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Formulae

Nonlinear equations

Lagrange Interpolating : $L_i = \frac{(x-x_1)(x-x_2)}{(x_i-x_1)(x_i-x_2)} \cdots \frac{(x-x_n)}{(x_i-x_n)}$; $f(x) = \sum_{i=1}^{n} L_i(x) f(x_i)$

Newton-Raphson Method : $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}, i = 0,1,2 \ldots.$

System of linear equations

Gauss-Seidel Iteration:

$$x_i^{(k+1)} = \frac{b_i - \sum_{j=1}^{i-1} a_{ij}\ x_j^{(k+1)} - \sum_{j=i+1}^{n} a_{ij}\ x_j^{(k)}}{a_{ii}}, \forall i = 1,2,3,\ldots,n.$$

Interpolation

Natural Cubic Spline:

$$\left.\begin{aligned} h_k &= x_{k+1} - x_k \\ d_k &= \frac{f_{k+1} - f_k}{h_k} \end{aligned}\right\}, k = 0,1,2,3,\ldots,n-1$$

$$b_k = 6(d_{k+1} - d_k), k = 0,1,2,3,\ldots,n-2,$$

When; $m_0 = 0, m_n = 0,$

$$h_k m_k + 2(h_k + h_{k+1}) m_{k+1} + h_{k+1} m_{k+2} = b_k, k = 0,1,2,3,\ldots,n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1} - x)^3 + \frac{m_{k+1}}{6h_k}(x - x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6} h_k\right)(x_{k+1} - x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6} h_k\right)(x - x_k) \quad , k = 0,1,2,3,\ldots n-1$$

Numerical Differentiation

2-point forward difference: $f'(x) \approx \frac{f(x+h) - f(x)}{h}$

2-point backward difference: $f'(x) \approx \frac{f(x) - f(x-h)}{h}$

3-point central difference: $f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$

3-point forward difference: $f'(x) \approx \frac{-f(x+2h) + 4f(x+h) - 3f(x)}{2h}$

3-point backward difference: $f'(x) \approx \frac{3f(x) - 4f(x-h) + f(x-2h)}{2h}$

5-point difference formula: $f'(x) \approx \frac{-f(x+2h) + 8f(x+h) - 8f(x-h) + f(x-2h)}{12h}$

3-point central difference: $f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$

5-point difference formula: $f''(x) \approx \frac{-f(x+2h) + 16f(x+h) - 30f(x) + 16f(x-h) - f(x-2h)}{12h^2}$

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Numerical Integration

Simpson $\frac{1}{3}$ Rule : $\int_a^b f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{\substack{i=1 \\ i\ odd}}^{n-1} f_i + 2\sum_{\substack{i=2 \\ i\ even}}^{n-2} f_i\right]$

Simpson $\frac{3}{8}$ Rule : $\int_a^b f(x)dx \approx \frac{3}{8}h\,[(f_0 + f_n) + 3(f_1 + f_2 + f_4 + f_5 + \cdots + f_{n-2} + f_{n-1}) + 2(f_3 + f_6 + \cdots + f_{n-3}]$

2-point Gauss Quadrature: $\int_a^b g(x)dx = \left[g\left(-\frac{1}{\sqrt{3}}\right) + g\left(\frac{1}{\sqrt{3}}\right)\right]$

3-point Gauss Quadrature: $\int_a^b g(x)dx = \left[\frac{5}{9}g\left(-\sqrt{\frac{3}{5}}\right) + \frac{8}{9}g(0) + \frac{5}{9}g\left(\sqrt{\frac{3}{5}}\right)\right]$

Eigen Value

Power Method : $v^{(k+1)} = \frac{1}{m_{k+1}}Av^{(k)}, k = 0,1,2 \ldots.$

Shifted Power Method :$v^{(k+1)} = \frac{1}{m_{k+1}}A_{shifted}v^{(k)}, k = 0,1,2 \ldots.$

Ordinary Differential Equation

Fourth-order Runge-Kutta Method : $y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

where $k_1 = hf(x_i, y_i)$ $\quad k_2 = hf(x_i + \frac{h}{2}, y_i + \frac{k_1}{2})$

$k_3 = hf(x_i + \frac{h}{2}, y_i + \frac{k_2}{2})$ $\quad k_4 = hf(x_i + h, y_i + k_3)$

Partial Differential Equation

Heat Equation: Finite Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j+1} - u_{i.j}}{k} = c^2\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$

Heat Equation: Crank-Nicolson Implicit Finite-Difference Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j+\frac{1}{2}} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j+\frac{1}{2}}$$

$$\frac{u_{i,j+1} - u_{i.j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{h^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$

Poisson Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial x^2}\right)_{i,j} + \left(\frac{\partial^2 u}{\partial y^2}\right)_{i,j} = f_{i,j} \qquad \frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{h^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{k^2} = 0$$

Wave Equation: Finite Difference Method

$$\left(\frac{\partial^2 u}{\partial t^2}\right)_{i,j} = \left(c^2\frac{\partial^2 u}{\partial x^2}\right)_{i,j} \qquad \frac{u_{i,j-1} - 2u_{i.j} + u_{i,j+1}}{k^2} = c^2\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}$$