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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	CALCULUS
COURSE CODE	:	BFC 15003
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JULY/ AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	1.	ANSWER ALL QUESTIONS
	2.	THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.**
	3.	STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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Q1 (a) For a function of $y = \frac{3}{4}x^4 - 3x^3$:

i) Find all the critical points.

(5 marks)

ii) Determine the intervals where the function is increasing, decreasing, concave up and concave down. Determine the inflection point, if any.

(9 marks)

(b) A police helicopter is flying at 200 kilometers per hour at a constant altitude of 1 km above a straight road as shown in **Figure Q1**. The pilot uses radar to determine that an oncoming car is at a distance of exactly 2 kilometers from the helicopter, and that this distance is decreasing at 300 kilometers per hour. Calculate the speed of the car.

(6 marks)

Q2 (a) Find the differentiation for the followings:

i) $y = \sin^3(10x^2 - 5)^4$

(3 marks)

ii) $y = (\sec x)(-\csc x + \cot x)$

(4 marks)

(b) Determine the $\frac{dy}{dx}$ of $x^4 + 8y^5 - \cos^2({}^{x}/_{y}) = 0$ by using implicit differentiation.

(7 marks)

(c) Given that $x = t - \sin t \cos t$ and $y = 4\cos t$. Determine $\frac{dy}{dx}$.

(6 marks)

Q3 (a) Find the integration for the followings:

i) $\int_1^6 (2x^5 - 3x^4 + 10x^2 - 7x + 5)dx$

(4 marks)

ii) $\int_0^{\pi} (7 + \sin x)dx$

(5 marks)

(b) Evaluate the following integrals using a suitable integration method.

i) $\int \frac{3x^3}{(10+x^4)^6}\,dx$

(6 marks)

ii) $\int x^6 \ln 2x\,dx$

(5 marks)

Q4 (a) Calculate the arc length of curve $x = r\cos t$ and $y = r\sin t$ within $0 \leq t \leq 2\pi$.

(5 marks)

(b) Determine the area of the region enclosed by $y = 5$ and $y = x^2 - 4$.

(5 marks)

(c) Evaluate the area of surface that is generated from the curve $y = x^2$ from (1,1) to (2,4) that is revolved about y-axis.

(10 marks)

Q5 (a) Determine $\frac{d^2y}{dx^2}$ as a function of t if $x = t - t^2$ and $y = t - t^3$.

(8 marks)

(b) Evaluate $\int \frac{15x+9}{x+5}\,dx$.

(6 marks)

(c) By using the substitution technique, $\int \frac{4}{x\sqrt{x^4-4}}\,dx$.

(6 marks)

- END OF QUESTIONS –

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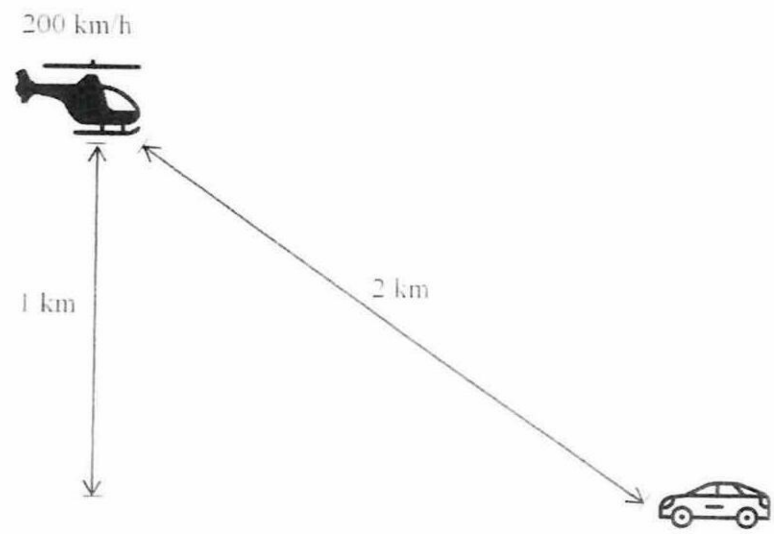

Figure Q1

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Formula

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$ stant	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n dx = \frac{x^{n+1}}{n+1}+C, \quad n\neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\int \sec x\tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\,x]=-\text{cosec}\,x\cot x$	$\int \text{cosec}\,x\cot x\,dx = -\text{cosec}\,x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\,x]=-\text{sech}\,x\tanh x$	$\int \text{sech}\,x\tanh x\,dx = -\text{sech}\,x + C$
$\frac{d}{dx}[\text{cosech}\,x]=-\text{cosech}\,x\coth x$	$\int \text{cosech}\,x\coth x\,dx = -\text{cosech}\,x + C$

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Formula

Trigonometric	**Hyperbolic**
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \csc^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	**Inverse Hyperbolic**
Logarithm	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2+1}\right)$, any x
$a^x = e^{x \ln a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2-1}\right)$, $x \ge 1$
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1+x}{1-x}\right)$, $-1 < x < 1$

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Formula

Integration of Inverse Functions
$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{\lvert a \rvert \sqrt{a^2 - x^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0$
$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, \quad x > a$
$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x \rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x \rvert > a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a$
$\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a$

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Formula

Differentiation of Inverse Functions	
y	$\dfrac{dy}{dx}$
$\sin^{-1} u$	$\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad \lvert u\rvert < 1$
$\cos^{-1} u$	$-\dfrac{1}{\sqrt{1-u^2}}\dfrac{du}{dx}, \quad \lvert u\rvert < 1$
$\tan^{-1} u$	$\dfrac{1}{1+u^2}\dfrac{du}{dx}$
$\cot^{-1} u$	$-\dfrac{1}{1+u^2}\dfrac{du}{dx}$
$\sec^{-1} u$	$\dfrac{1}{\lvert u\rvert\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u\rvert > 1$
$\operatorname{cosec}^{-1} u$	$-\dfrac{1}{\lvert u\rvert\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u\rvert > 1$
$\sinh^{-1} u$	$\dfrac{1}{\sqrt{u^2+1}}\dfrac{du}{dx}$
$\cosh^{-1} u$	$\dfrac{1}{\sqrt{u^2-1}}\dfrac{du}{dx}, \quad \lvert u\rvert > 1$
$\tanh^{-1} u$	$\dfrac{1}{1-u^2}\dfrac{du}{dx}, \quad \lvert u\rvert < 1$
$\coth^{-1} u$	$-\dfrac{1}{1-u^2}\dfrac{du}{dx}, \quad \lvert u\rvert > 1$
$\operatorname{sech}^{-1} u$	$-\dfrac{1}{u\sqrt{1-u^2}}\dfrac{du}{dx}, \quad 0 < u < 1$
$\operatorname{cosech}^{-1} u$	$-\dfrac{1}{\lvert u\rvert\sqrt{1+u^2}}\dfrac{du}{dx}, \quad u \neq 0$

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Formula

Area between two curves

Case 1- Integrating with respect to x: $A = \int_a^b [f(x) - g(x)]dx$

Case 2- Integrating with respect to y: $A = \int_c^d [f(y) - g(y)]dy$

Area of surface of revolution

Case 1- Revolving the portion of the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

Case 2- Revolving the portion of the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve- Revolving the curve about x-axis: $S = 2\pi \int_a^b y\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Parametric curve- Revolving the curve about y-axis: $S = 2\pi \int_c^d x\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Arc length

x-axis: $L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

y-axis: $L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

Parametric curve: $L = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

Curvature

Curvature, $\mathrm{K} = \dfrac{\left|\frac{d^2y}{dx^2}\right|}{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\mathrm{K}}$

Curvature of parametric curve

Curvature, $\mathrm{K} = \dfrac{|\dot{x}\ddot{y} - \dot{y}\ddot{x}|}{(\dot{x}^2 + \dot{y}^2)^{3/2}}$

Radius of curvature, $\rho = \dfrac{1}{\mathrm{K}}$