UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	DIFFERENTIAL EQUATIONS
COURSE CODE	:	BWC 11103 / BWC 10603
PROGRAMME CODE	:	BWC
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 Solve the following differential equations

(a) $x\dfrac{dy}{dx}-y=\dfrac{x}{x+1}, \quad y(1)=0.$

(12 marks)

(b) $\left(y\cos x+2xe^{y}\right)+\left(\sin x+x^{2}e^{y}+2\right)y'=0.$

(8 marks)

Q2 (a) Find the solution of differential equations

$$2y''-6y'-2y=0.$$

(3 marks)

(b) Find the charge on the capacitor in an LRC series circuit as shown in **Figure Q2.1** when, L = 0.5 H, R = 20 Ω, C = 0.001 F, and $E(t)$ = 100 sin $60t$ V.

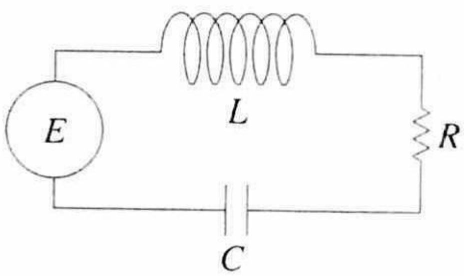

Figure Q2.1

(9 marks)

(c) Referring to the **Table 1 APPENDIX A.1**, evaluate $\mathcal{L}^{-1}\{F(s)\}$ for

$$F(s)=\frac{s+3}{4s^{2}+9}.$$

(8 marks)

Q3 (a) Solve the following first-order initial value problem (IVP) in three decimal places by second-order Taylor series method with step size, h = 0.2.

$$y'=\cos 2x+\sin 3x, \quad 0\le x\le 1, \quad y(0)=1.$$

Given the exact solution is

$$y(x)=\frac{1}{2}\sin 2x-\frac{1}{3}\cos 3x+\frac{4}{3}.$$

Find its errors. Refer **Formula 1 APPENDIX B.1**.

(10 marks)

(b) Solve

$$y'' - y = x^2,$$

with

$$y(0) = 0, y(1) = 0,$$

using finite difference method approach in four decimal places with $h = 0.25$. Refer **Formula 2 APPENDIX C.1**.

(10 marks)

Q4 (a) The temperature distribution $u(x,t)$ of a one-dimensional rod is governed by the heat equation

$$\frac{\partial u}{\partial t} = \sqrt{2}\frac{\partial^2 u}{\partial x^2},$$

Given the initial condition,

$$u(x,0) = \sin x, \quad 0 \le x \le \pi,$$

and boundary conditions are

$$u(0,t) = 0, \quad u(\pi,t) = 0, \quad t > 0.$$

Find the solution of heat flow problem using separation of variable method.

(8 marks)

(b) A vibrating string of unit length is fixed at both ends is subjected to the initial conditions

$$u(x,0) = x(1-x), \quad \frac{\partial u}{\partial t}(x,0) = 2x^2, \quad 0 \le x \le 1.$$

Solve $u(x,t)$,where the governing equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad 0 \le x \le 1, \quad t \ge 0,$$

for $0 \le t \le 0.3$ using $h = \Delta x = 0.2$ and $k = \Delta t = 0.1$ by explicit finite-difference method in four decimal places. Refer **Formula 3 APPENDIX D.1**.

(12 marks)

- END OF QUESTIONS -

APPENDIX A
Laplace Transform

Table 1 APPENDIX A.1

$y(t)$	$\mathcal{L}\{y(t)\} = F(t)$
1	$\frac{1}{s}$
t^n	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sin at$	$\frac{a}{s^2+a^2}$

APPENDIX B
Second-Order Taylor Series Method

Formula 1 APPENDIX B.1

$$y_{i+1} = y_1 + hy_i' + \frac{h^2}{2!} y_i''$$

APPENDIX C
Boundary Value Problem: Finite-Difference Method

Formula 2 APPENDIX C.1

$$y_i' \approx \frac{y_{i+1} - y_{i-1}}{2h}$$

$$y_i'' \approx \frac{y_{i-1} - 2y_i + y_{i-1}}{h^2}$$

APPENDIX D
Wave Equation: Explicit Finite-Difference Method

Formula 3 APPENDIX D.1

$$\frac{\partial^2 u_{i,j}}{\partial t^2} = c^2 \frac{\partial^2 u_{i,j}}{\partial x^2}$$

$$\frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{k^2} = c^2 \left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2} \right)$$