Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2022/2023

COURSE NAME	:	ACTUARIAL MATHEMATICS II
COURSE CODE	:	BWA 31503
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JULY / AUGUST 2023
DURATION	:	2 HOURS 30 MINUTES
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA ☐ Open book ☒ Closed book 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) Cash value is the amount available in cash upon cancellation of an insurance policy.

(i) Identify **THREE (3)** common insurance options that use the policy's net cash values.
(3 marks)

(ii) Outline the similarities or differences among these **THREE (3)** insurance options given in **Q1(a)(i)**.

(7 marks)

(b) You are given the Illustrative Life Table **(Table Q1(b))** with interest rate of 6%. A 40-year-old male purchased the 10-year endowment insurance where the net level premium is RM 81.36 and the death benefit is RM 1,000. Estimate the terminal reserve for the fourth year of a 40-year old male.

Table Q1(b)

Age	Number Alive	Number That Die
40	937.72	2.83
41	934.89	3.08
42	931.81	3.32
43	928.50	3.59
44	924.90	3.88

(10 marks)

Q2 (a) Consider a policy issued at age 35 with an initial gross premium of 1,000 and initial benefit of 120,000. Use the Illustrative Life Table in **Table Q2(a)** with 6% interest to analyze the excess first-year expense allowance and the fifth-year reserve.

Table Q2(a)

Age	l_x	d_x	$1{,}000q_x$
30	95 013.79	145.2682	1.5289
31	94 868.53	152.6317	1.6089
32	94 715.89	160.6896	1.6965
33	94 555.20	169.5052	1.7927
34	94 385.70	179.1475	1.8980
35	94 206.55	189.6914	2.0136
36	94 016.86	201.2179	2.1402
37	93 815.64	213.8149	2.2791
38	93 601.83	227.5775	2.4313
39	93 374.25	242.6085	2.5982
40	93 131.64	259.0186	2.7812
41	92 872.62	276.9271	2.9818
42	92 595.70	296.4623	3.2017
43	92 299.23	317.7619	3.4427
44	91 981.47	340.9730	3.7070
45	91 640.50	366.2529	3.9966

(10 marks)

(b) Five years after issue, the policyholder in **Q2(a)** wishes to change the policy to Term Insurance to age 65 with a coverage of 150,000. Determine the contract premium after the change.

(10 marks)

Q3 (a) Ahmad's age is 50.5 at the valuation date. He receives RM6,000 in salary in the month to the valuation date. Ahmad's salary increases yearly on 1 January and he is planning to retire at age 65. Assume the replacement ratio is 65% and the valuation date of 1 September. Using **Table Q3(a),**

Table Q3(a)

Age	s_x	x	s_x
30	1.00	50	3.41
31	1.06	51	3.63
32	1.13	52	3.86
33	1.20	53	4.10
34	1.28	54	4.35
35	1.36	55	4.62
36	1.44	56	4.91
37	1.54	57	5.21
38	1.63	58	5.53
39	1.74	59	5.86
40	1.85	60	6.21
41	1.96	61	6.56
42	20.9	62	6.93
43	2.22	63	7.31
44	2.36	64	7.70
45	2.51	65	8.08
46	2.67	66	8.48
47	2.84	67	8.91
48	3.02	68	9.35
49	3.21	69	9.82

(i) determine the salary that Ahmad receives over the year of age $\left(49\frac{5}{6}, 50\frac{5}{6}\right)$,

(1 mark)

(ii) calculate the expected salary in Ahmad's final year of work,

(5 marks)

(iii) calculate Ahmad's target pension benefit per year.

(3 marks)

(b) Suppose Fasha, aged 30, is a newly hired employee of DRB Group. She receives RM80,000 in her first year of service at the company. Assuming

- Fasha's salary increases 3% per year,
- she receives merit increases of 5% at each of the first three (3) employment anniversaries,
- the pension benefit formula is 1% of the final five (5) year average salary per year of service.

(i) If Fasha retires at age 65, predict the projected final **FIVE (5)** year average salary.
(5 marks)

(ii) Forecast the projected pension benefit Fasha will receive at age 65.
(2 marks)

(iii) Compute the employee's replacement ratio.
(4 marks)

Q4 A Lexis diagram provides a convenient way of showing the relationship between periods and cohorts. Demographic events can be viewed either by calendar time, age or cohort.

(a) Using the Lexis diagram in **Figure Q4(a)**, calculate

(i) the age difference between the oldest and the youngest employees at time -25,
(3 marks)

(ii) the number of employees who have attained age 35 while active in the workforce,
(1 mark)

(iii) the number of employees at time -25, who have attained or will attain age 50 while in the workforce.
(2 marks)

(b) The generation force of mortality at age x for those born at time u is denoted by

$$\mu(x,u) = -\frac{1}{l(x,u)}\frac{\partial}{\partial x} l(x,u).$$

From **Figure Q4(b)**, use the double integral method to show that the number of lives that will attain age x_0 between times t_0 and $t_0 + 1$ and die before time $t_0 + 3$ is given by

$$\int_{t_0}^{t_0+1} l(x_0, y - x_0)\,dy - \int_{x_0+2}^{x_0+3} l(w, t_0 + 3 - w)\,dw.$$

(8 marks)

(c) A population density function is defined by

$$l(x,u)=b(u)s(x,u).$$

Let

$$b(u)=100\left[1-e^{-u/100}\right] \quad u>0$$
$$s(x)=e^{-x/100} \quad x>0.$$

Compute the number of individuals between ages 25 and 50 at time 100.

(6 marks)

-END OF QUESTIONS-

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2022 /2023 PROGRAMME CODE : BWA
COURSE NAME : ACTUARIAL MATHEMATICS II COURSE CODE : BWA 31503

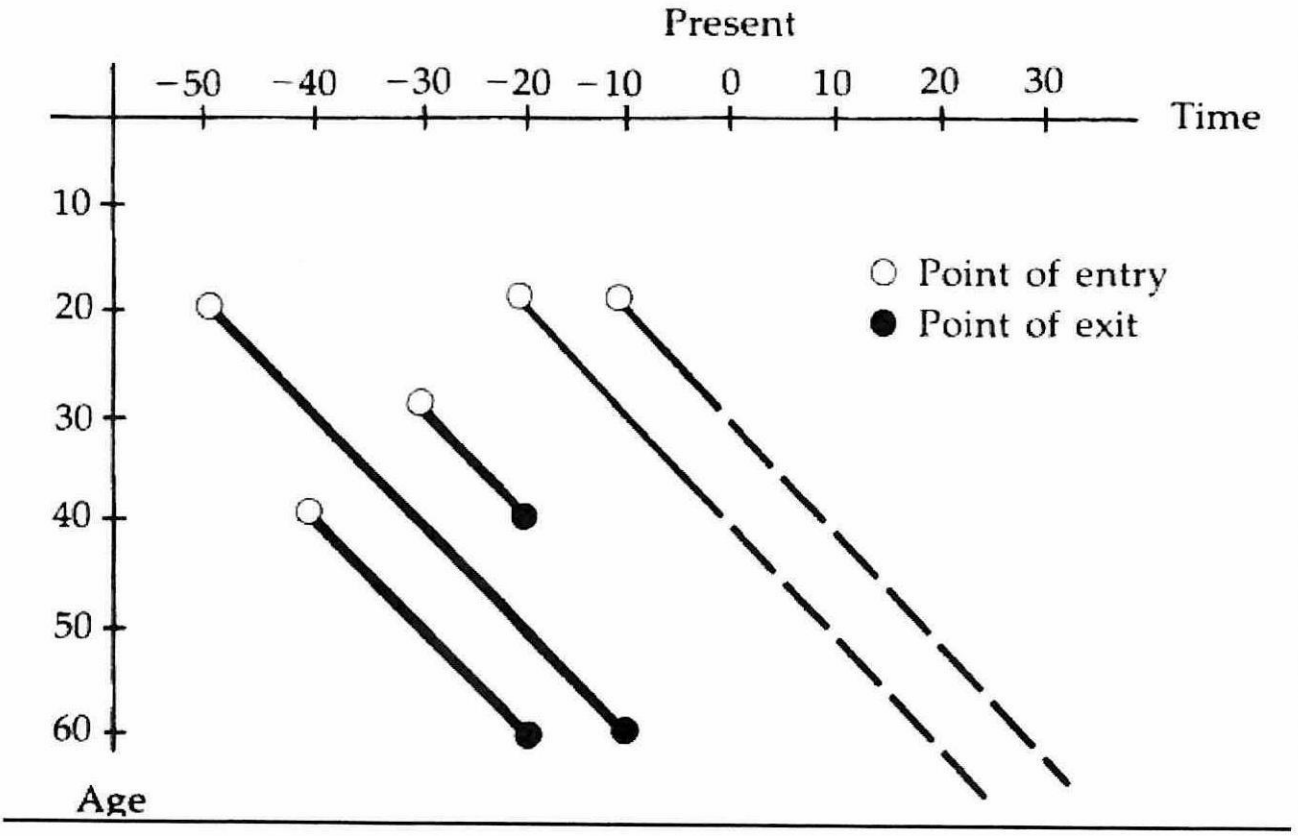

Figure Q4(a)

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2022/2023
COURSE NAME : ACTUARIAL MATHEMATICS II
PROGRAMME CODE : BWA
COURSE CODE : BWA 31503

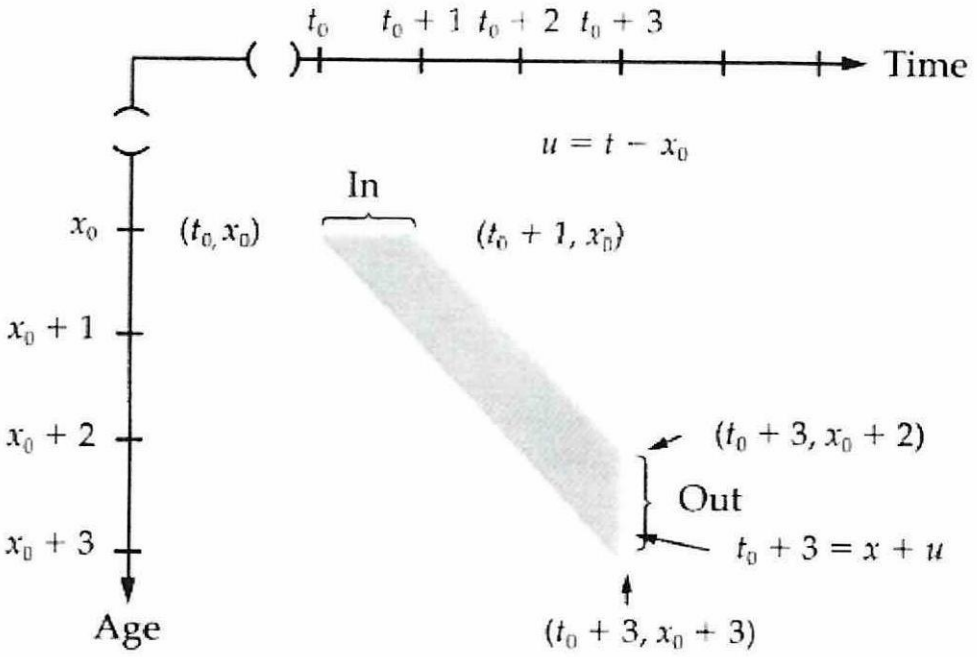

Figure Q4(b)

FINAL EXAMINATION

SEMESTER / SESSION : SEM II / 2022/2023
COURSE NAME : ACTUARIAL MATHEMATICS II
PROGRAMME CODE : BWA
COURSE CODE : BWA 31503

FORMULAE

$$\Pr(N=n)=\frac{\lambda^n e^{-\lambda}}{n!}$$

$$\ddot{a}_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^k\cdot {}_kp_x$$

$$A^1_{x:\overline{n}|}=\sum_{k=0}^{n-1}\upsilon^{k+1}\cdot {}_kp_x\cdot q_{x+k}$$

$${}_nE_x=v^n\,{}_nP_x$$

$$v^n=\frac{1}{(1+i)^n}$$

$${}_kP_x=\frac{l_{40+k}}{l_{40}}$$

$$-{}_0V=P-\upsilon q_x b$$

$${}_kV=\frac{{}_0V+P\ddot{a}_{x:\overline{k}|}-bA^1_{x:\overline{k}|}}{{}_kE_x}$$

$${}_{k+g}V'=\frac{{}_kV'+P'\ddot{a}_{x+k:\overline{g}|}-b'A^1_{x+k:\overline{g}|}}{{}_gE_{x+k}}$$

$$\int_{t_0}^{t_1} l(x,t-x)\,dt$$

$$\int_{x_0}^{x_1} l(x,t_0-x)\,dx$$