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Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2014/2015

COURSE NAME	:	TECHNICAL MATHEMATICS I
COURSE CODE	:	DAS11003
PROGRAMME	:	1 DAB/ 1 DAJ/ 1 DAR/ 1 DAK
EXAMINATION DATE	:	DECEMBER 2014/ JANUARY 2015
DURATION	:	3 HOURS
INSTRUCTION	:	**A)** ANSWER **ALL** QUESTION IN A B) ANSWER **THREE (3)** QUESTIONS IN B

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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SECTION A

Q1 (a) Given

$$A=\begin{pmatrix}1 & -3 & -1\\ 0 & 3 & 5\\ 2 & 4 & 6\end{pmatrix}, B=\begin{pmatrix}-1 & 3 & 9\\ -1 & 0 & 1\\ -2 & 3 & 4\end{pmatrix}, C=\begin{pmatrix}3 & 0\\ -1 & 2\\ 4 & 1\end{pmatrix}, D=\begin{pmatrix}-1 & -1\\ 0 & 0\\ 3 & 5\end{pmatrix}$$

Calculate

(i) $A+3B$.

(3 marks)

(ii) $CD^T - B$.

(3 marks)

(iii) AD.

(3 marks)

(iv) $-(D-5C)^T$.

(4 marks)

(b) A linear equation system is given as below:

$$2x+3y+4z=2$$
$$x-4y+3z=2$$
$$5x+y+z=-4$$

(i) Write into matrix equation, $AX=B$.

(1 mark)

(ii) Find the determinant A, adjoin A and inverse matrix of A.

(8 marks)

(iii) Solve x, y and z.

(3 marks)

SECTION B

Q2 (a) (i) Find the value of x and y if $x + 6y = 0$ and $2x + 4y = 8$.

(3 marks)

(ii) Given $4^x \cdot 64^{2y} = 1$ and $3^{2x} \cdot 81^y = \frac{1}{27}$. Find the values of x and y.

(6 marks)

(b) (i) Calculate $\left(\frac{4+\sqrt{5}}{3-\sqrt{8}}\right) \div \left(\frac{4+\sqrt{5}}{2-\sqrt{3}}\right)$

(4 marks)

(ii) Given $x = \frac{4}{2-\sqrt{3}}$ and $y = \frac{3+\sqrt{2}}{5}$. Calculate $\frac{5}{4}xy - 10y - x$ and simplify the answer by rationalize the denominator.

(5 marks)

(c) (i) Simplify $\log_2\left(\frac{64 \times 8^x}{16^y}\right)$.

(4 marks)

(ii) Calculate $x + y$ if given $\log_x 64 = 6$ and $\log_4 y = 4$.

(3 marks)

Q3 (a) Find the root of x of the function $f(x) = x^3 - 2x^2 + 5x + 6$ between $[-1, -0.5]$ by using Bisection method. Iterate until. $|f(x)_i| \leq \varepsilon = 0.005$.

(7 marks)

(b) If $A = 3x^3 + 4x - 9$ and $B = x^3 - 2x^2 + 6x - 3$. Find the root of x of $A - 3B - 5$.

(6 marks)

(c) Solve $x^2(x+2) \geq 3x^2 + 6x$

(6 marks)

(d) Express $\frac{6x^2 - 9x + 9}{x^2(x-3)}$ into partial fraction.

(6 marks)

Q4 (a) The sum of the first 11 terms of an arithmetic sequence is 110 and the sum of the first 20 terms is 290. Find the 11^{th} and 20^{th} terms of the sequence

(9 marks)

(b) Initially, a pendulum swings through an arc of 18 inches length. On each successive swing, the length of the arc is 98% of the previous length.

(i) Find the length of the 10^{th} swing.

(5 marks)

(ii) Find the total distance after 10^{th} swing.

(2 marks)

(iii) Evaluate the total distance that the pendulum have swung until it stop.

(2 marks)

(c) (i) Expand $(x+3)^5$

(5 marks)

(ii) Find the 7^{th} term of $(2x+y)^9$

(2 marks)

Q5 (a) Prove that $\sin\theta + \tan\theta = \tan\theta(1+\cos\theta)$

(5 marks)

(b) Solve in between 0 and π for each expression.

(i) $\sin 2A = 0.5$

(4 marks)

(ii) $\cos 2B = 0.5$

(4 marks)

(c) If $\cos\theta = \dfrac{2}{3}$ and θ in the first quadrant. Evaluate

(i) $\tan\dfrac{\theta}{2}$

(5 marks)

(ii) $\sin 2\theta$

(3 marks)

(iii) $\cos 4\theta$

(4 marks)

Q6 (a) Calculate

(i) $\begin{pmatrix} 1 & 2 & 4 \\ 2 & 0 & -3 \\ 3 & -1 & 5 \end{pmatrix} - \begin{pmatrix} -3 & 4 & 2 \\ 5 & -5 & -4 \\ 0 & 7 & 1 \end{pmatrix}$

(2 marks)

(ii) $5\begin{pmatrix} 1 & 2 & 3 \\ 2 & -2 & 4 \\ 5 & -3 & 1 \end{pmatrix} + \frac{3}{2}\begin{pmatrix} 2 & 4 & 0 \\ -2 & 0 & 2 \\ 8 & -4 & -4 \end{pmatrix}$

(2 marks)

(iii) $\begin{pmatrix} 1 & 0 & -2 \\ 3 & -1 & 4 \\ 5 & 2 & 7 \end{pmatrix}\begin{pmatrix} 2 & -12 \\ 5 & 4 \\ 10 & -6 \end{pmatrix}$

(2 marks)

(iv) $\begin{pmatrix} 2 & 1 & 3 \\ 4 & -2 & 5 \end{pmatrix}\begin{pmatrix} 3 & 5 \\ 2 & -1 \\ 6 & 4 \end{pmatrix} - \begin{pmatrix} 10 & 4 \\ 3 & 5 \end{pmatrix}^T$

(2 marks)

(b) Given a system of linear equation:

$$x + y + z = 5$$
$$2x - y + z = 12$$
$$x + 3y + z = 1$$

(i) Write into matrix equation, AX = B.

(3 marks)

(ii) Write the augmented matrix, [A|B].

(2 marks)

(iii) By using Gauss-Jordan elimination method, find x, y and z starting with the following row-operations:
R2 – R3, R2 – R1, R3 – R1, ...

(12 marks)

- END OF QUESTION -

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Exponent, Radical & Logarithms

i) $x^m . x^n = x^{m+n}$

ii) $\dfrac{x^m}{x^n} = x^{m-n}$

iii) $(x^m)^n = x^{mn}$

iv) $x^{p/q} = \left(\sqrt[q]{x}\right)^p$

v) $x = b^n \Leftrightarrow \log_b x = n$

vi) $\log_b(xy) = \log_b x + \log_b y$

vii) $\log_b\left(\dfrac{x}{y}\right) = \log_b x - \log_b y$

viii) $\log_b x^k = k \log_b x$

ix) $\log_a x = \dfrac{\log_b x}{\log_b a}$

Polynomial

i) $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

ii)
$$x^2 + bx + c = x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$
$$= \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$$

iii) $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$

Sequence & Series

i) $\sum_{k=1}^{n} c = cn$

ii) $\sum_{k=1}^{n} k = \dfrac{n(n+1)}{2}$

iii)
$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

Arithmetic Series

i) $T_n = a + (n-1)d$
$d = u_n - u_{n-1}$

ii) $S_n = \dfrac{n}{2}(a + u_n)$

iii) $S_n = \dfrac{n}{2}[2a + (n-1)d]$

Geometric Series

i) $T_n = ar^{n-1}$

ii) $r = \dfrac{u_n}{u_{n-1}}$

iii) $S_n = \dfrac{a(1-r^n)}{1-r}$

iv) $S_\infty = \dfrac{a}{1-r}$

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The Binomial Theorem

i) $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

ii) $(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \ldots + \binom{n}{n}b^n$

iii) $(r+1)th = \binom{n}{r}a^{n-r}b^r$

Trigonometric Identity

i) $\cos^2\theta + \sin^2\theta = 1$

ii) $1 + \tan^2\theta = \sec^2\theta$

iii) $\cot^2\theta + 1 = \csc^2\theta$

Addition and Subtraction Formulas:

i) $\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

ii) $\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

iii) $\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

Double - Angle Formulas

i) $\sin 2\theta = 2\sin\theta\cos\theta$

ii) $\cos 2\theta = \cos^2\theta - \sin^2\theta$
OR $\cos 2\theta = 2\cos^2\theta - 1$
OR $\cos 2\theta = 1 - 2\sin^2\theta$

iii) $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Half – Angle Formulas

i) $\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

ii) $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

iii) $\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

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Trigonometry Equation in the Form: $a \sin \theta + b \cos \theta = c$

Let
$$\begin{aligned} a\sin\theta + b\cos\theta &= r\sin(\theta+\alpha) \\ &= r(\sin\theta\cos\alpha + \cos\theta\sin\alpha) \\ &= (r\cos\alpha)\sin\theta + (r\sin\alpha)\cos\theta \end{aligned}$$

We get $a = r\cos\alpha$ and $b = r\sin\alpha$ $\Rightarrow r = \sqrt{a^2+b^2}$ $\quad \alpha = \tan^{-1}\left(\frac{b}{a}\right)$

We use the above to solve:

$$a\sin\theta + b\cos\theta = r\sin(\theta+\alpha)$$
$$a\sin\theta - b\cos\theta = r\sin(\theta-\alpha)$$
$$a\cos\theta + b\sin\theta = r\cos(\theta-\alpha)$$
$$a\cos\theta - b\sin\theta = r\cos(\theta+\alpha)$$

Matrices

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}, \quad |A| = a_{11}\begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix} - a_{12}\begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix} + a_{13}\begin{vmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{vmatrix}$$

$$Adj(A) = (c_{ij})^T$$

$$A^{-1} = \frac{1}{|A|} Adj(A)$$