UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	ACTUARIAL MATHEMATICS I
COURSE CODE	:	BWA 31403
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

TERBUKA

Q1 (a) Let $F_0(t) = 1 - (1 - t/105)^{1/5}$ for $0 \le t \le 105$. Calculate

(i) the probability that a newborn life dies before age 60,

(2 marks)

(ii) the probability that a life age 20 dies between ages 90 and 100.

(2 marks)

(b) Actuarial Science has developed its own notation for survival and mortality probabilities. Formulate the meaning of the following notations

(i) ${}_5p_{20}$,

(2 marks)

(ii) ${}_{20|5}q_{20}$,

(2 marks)

(iii) ${}_{5|2}q_{[20]+1}$.

(2 marks)

(b) Mortality for a population consisting of females and males follows a select and ultimate table, an extract of which is given in **Table Q1(b)**. Males have a 3-year select period while females have a 2-year select period. Assume mortality follows the Uniform Distribution of Death (UDD) between integral ages.

Table Q1(b): Extract of select and ultimate table

Males						Females				
x	$l_{[x]}$	$l_{[x]+1}$	$l_{[x]+2}$	l_{x+3}	$x+3$	x	$l_{[x]}$	$l_{[x]+1}$	l_{x+2}	$x+2$
50	80960	79827	78522	77025	53	50	70764	69124	67224	52
51	79530	78334	76958	75382	54	51	68823	67118	65146	53
52	78021	76760	75312	73655	55	52	66805	65036	62993	54
53	76430	75103	73581	71842	56	53	64711	62879	60768	55
54	74756	73362	71765	69944	57	54	62544	60651	58475	56
55	72998	71535	69863	67958	58	55	60305	58354	56117	57

(i) Propose the probability that a randomly chosen female from this population now age 51.25, with select age 50, will die within the next 3 years and 9 months.

(7 marks)

(ii) Propose the probability that a randomly chosen male from this population, at select age 51, will survive within the ages of 52.35 and 56.75.

(8 marks)

Q2 (a) **Table Q2(a)** shows the survival probabilities, p_{x+t} at time, t of a certain population with assumption $i = 4\%$.

Table Q2(a): Survival probabilities

t	0	1	2	3	4	5	6
p_{x+t}	0.8	0.9	0.95	0.96	0.95	0.95	0.9

(i) If death occurs in the first policy year, calculate the actuarial present value (APV) of a term life insurance with a benefit of RM1 issued to (x), payable at the end of the first policy year if death occurs in the first policy year.

(4 marks)

(ii) Calculate the actuarial present value (APV) of a term life insurance with a benefit of RM3 issued to $(x+1)$, payable at the end of the third policy year if death occurs in the third policy year.

(4 marks)

(iii) Determine the actuarial present value (APV) of an endowment insurance issued to $(x + 2)$, which pays RM1 at the end of the year of death if deaths occur in the first two years, and RM2 at the end of the second policy year if the life survives.

(4 marks)

(iv) Explain why your answer to **Q2(a)(iii)** is greater than your answer to **Q2(a)(ii)**.

(2 marks)

(b) Ismail aged 45, Chong aged 44 and Muthu aged 43 buy a whole life insurance policy on the day of their birthdays. Their policies will pay RM 50,000 at the end of the year of death. The actuarial present value for Ismail, Chong and Muthu is RM 25,000, RM23,702 and RM21,221, respectively. Suppose that $i = 0.06$.

(i) Predict the probability that Chong will die within one year.

(4 marks)

(ii) Predict the probability that Muthu will die within one year.

(4 marks)

(iii) Analyze answer in **Q2(b)(i)** and **Q2(b)(ii)**.

(3 marks)

Q3 (a) **Table Q3(a)** shows annuity payment and the survival probabilities, p_{x+t} at time, t for a special 3-year temporary life annuity-due.

Table Q3(a): Annuity payment dan survival probabilities

t	Annuity Payment	p_{x+t}
0	15	0.95
1	20	0.90
2	25	0.85

Given $i = 0.06$, calculate

(i) the present value random variable Y for the annuity if the annuitant dies in the first and second year.

(4 marks)

(ii) the present value random variable Y for the third year. (Hint: Use answers in **Q3(a)(i)**).

(2 marks)

(iii) probability q_x if (x) dies in the first, second and third year.

(6 marks)

(iv) the expected present value $E(Y)$ of the present value random variable for this annuity

(2 marks)

(v) the variance of the present value random variable for this annuity.

(6 marks)

(b) The net premiums are determined by two factors that are beyond the control of an insurance company. Identify these two factors.

(4 marks)

Q4 (a) Consider the following five premiums.

$$P^{(2)}\left(\overline{A}_{40:\overline{25}|}\right),\ \overline{P}\left(\overline{A}_{40:\overline{25}|}\right),\ P^{\{4\}}\left(\overline{A}_{40:\overline{25}|}\right),\ P\left(\overline{A}_{40:\overline{25}|}\right),\ P^{\{12\}}\left(\overline{A}_{40:\overline{25}|}\right).$$

(i) Arrange the premiums in order of magnitude.

(5 marks)

(ii) Indicate your reasoning to answer in **Q4(a)(i)**.

(5 marks)

(b) Given $\dfrac{P^{1(12)}_{x:\overline{20|}}}{P^{1}_{x:\overline{20|}}} = 1.032$ and $P_{x:\overline{20|}} = 0.040$. Evaluate $P^{(12)}_{x:\overline{20|}}$.

(6 marks)

(c) For a 10-year term life insurance policy issued to a policyholder aged 50, you are given:

- The death benefit of RM 100 is payable at the end of the year of death,
- A level premium is paid at the beginning of each year during the term of the policy,
- Mortality follows the Illustrative Life Table in **Table Q4(d),**
- $i = 0.06$,
- Net premium is calculated according to the actuarial equivalence principle.

Using the Illustrative Life Table as shown in **Table Q4(c)**, calculate the net premium reserve at the end of year 5.

Table Q4(c): Illustrative life table

Age	$\ddot{a}_x$	$1000A_x$	$1000\left({}^2A_x\right)$	$1000\,{}_5E_x$	$1000\,{}_{10}E_x$
50	13.2668	249.05	94.76	721.37	510.81
51	13.0803	259.61	101.15	719.17	506.78
52	12.8879	270.50	107.92	716.76	502.40
53	12.6896	281.72	115.09	714.12	497.64
54	12.4856	293.27	122.67	711.24	492.47
55	12.2758	305.14	130.67	708.10	486.86
56	12.0604	317.33	139.11	704.67	480.79
57	11.8395	329.84	147.99	700.93	474.22
58	11.6133	342.65	157.33	696.85	467.12
59	11.3818	355.75	167.13	692.41	459.46
60	11.1454	369.13	177.41	687.56	451.20

(10 marks)

–END OF QUESTIONS–

FINAL EXAMINATION

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LIST OF FORMULAS

$$F_x(t) = \frac{F_0(x+t) - F_0(x)}{S_0(x)} \qquad S_0(x+t) = S_0(x) S_x(t) \qquad \mu_{x+t} = -S_x'(t) / S_x(t)$$

$${}_t p_x = l_{x+t}/l_x \qquad d_x = l_x - l_{x+1} \qquad d_x = l_x q_x \qquad {}_{t|u}q_x = {}_t p_x \cdot {}_u q_{x+t} \qquad d = i\upsilon$$

$$A_{x:\overline{n}|}^{\;\;1} = {}_n E_x = \upsilon^n {}_n p_x \qquad A_{x:\overline{n}|}^{\;\;1} = \int_n^\infty \upsilon^n \, {}_t p_x \, \mu_{x+t} \, dt \qquad \overline{A}_{x:\overline{n}|}^{1} = \int_0^n \upsilon^t \, {}_t p_x \, \mu_{x+t} \, dt \qquad \overline{A}_x = \int_0^\infty \upsilon^t \, {}_t p_x \, \mu_{x+t} \, dt$$

$$A_x = \sum_{k=0}^{\infty} \upsilon^{k+1} \, {}_k p_x \, q_{x+k} \qquad A_{x:\overline{n}|}^{1} = \sum_{k=0}^{n-1} \upsilon^{k+1} \, {}_k p_x \, q_{x+k} \qquad {}_{u|}\overline{A}_x = \int_u^\infty \upsilon^t \, {}_t p_x \, \mu_{x+t} \, dt = \overline{A}_x - \overline{A}_{x:\overline{u}|}^{1}$$

$$\overline{A}_{x:\overline{n}|} = \overline{A}_{x:\overline{n}|}^{1} + \overline{A}_{x:\overline{n}|}^{\;\;1} \qquad A_{x:\overline{n}|} = A_{x:\overline{n}|}^{1} + A_{x:\overline{n}|}^{\;\;1} \qquad {}_{u|}\overline{A}_{x:\overline{n}|}^{1} = \int_u^{u+n} \upsilon^t \, {}_t p_x \, \mu_{x+t} \, dt = \overline{A}_{x:\overline{u+n}|}^{1} - \overline{A}_{x:\overline{u}|}^{1}$$

$$A_{x:\overline{n}|}^{1} = A_x - \upsilon^n \, {}_n p_x \, A_{x+n} \qquad A_x = \upsilon q_x + \upsilon p_x A_{x+1} \qquad \overline{A}_x = \frac{i}{\delta} A_x$$

$$\overline{a}_x = \frac{1 - \overline{A}_x}{\delta} \qquad \overline{a}_x == \int_0^\infty \upsilon^t \, {}_t p_x \, dt \qquad \overline{a}_{x:\overline{n}|} = \frac{1 - \overline{A}_{x:\overline{n}|}}{\delta} \qquad \overline{a}_{x:\overline{n}|} = \int_0^n \upsilon^t \, {}_t p_x \, dt$$

$$\ddot{a}_x = \frac{1 - A_x}{d} \qquad \ddot{a}_x = \sum_{k=0}^{\infty} \upsilon^k \, {}_k p_x \qquad \ddot{a}_{x:\overline{n}|} = \frac{1 - A_{x:\overline{n}|}}{d} \qquad \ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} \upsilon^k \, {}_k p_x$$

$$a_x = \sum_{k=1}^{\infty} \upsilon^k \, {}_k p_x \qquad a_x = \ddot{a}_x - 1 \qquad a_{x:\overline{n}|} = \sum_{k=1}^{n} \upsilon^k \, {}_k p_x \qquad \ddot{a}_{x:\overline{n}|} - a_{x:\overline{n}|} = 1 - \upsilon^n \, {}_n p_x$$

$${}_{n|}\overline{a}_x = \int_n^\infty \upsilon^t \, {}_t p_x \, dt \qquad {}_{n|}\overline{a}_x = {}_n E_x \, \overline{a}_{x+n} \qquad {}_{n|}\ddot{a}_x = \sum_{k=n}^{\infty} \upsilon^k \, {}_k p_x \qquad {}_{n|}\ddot{a}_x = {}_n E_x \, \ddot{a}_{x+n}$$

$$P_x = \frac{A_x}{\ddot{a}_x} \qquad P_{x:\overline{n}|}^{1} = \frac{A_{x:\overline{n}|}^{1}}{\ddot{a}_{x:\overline{n}|}} \qquad P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \qquad P_{x:\overline{n}|}^{\;\;1} = \frac{A_{x:\overline{n}|}^{\;\;1}}{\ddot{a}_{x:\overline{n}|}}$$