Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	CALCULUS
COURSE CODE	:	BEE 10103
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS
		2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.**
		3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

Q1 (a) Evaluate the integration of the following functions:

(i) $\int \frac{x^2+2}{\sqrt[3]{x}} dx$

(2 marks)

(ii) $\int x^4 \, e^{x^5} dx$

(3 marks)

(iii) $\int t \operatorname{sech}(4t^2 + 5) \tanh(4t^2 + 5) dt$

(3 marks)

(b) Evaluate the following functions using integration by u-substitution.

(i) $\int (5x^4 - 1) e^{(x^5 - x)} dx$

(2 marks)

(ii) $\int \frac{1}{\sqrt{x}} sin \sqrt{x} \; dx$

(5 marks)

(c) Evaluate the following functions using integration by parts.

(i) $\int x^4 \ln x \, dx$

(5 marks)

(ii) $\int e^x \cos x \, dx$

(5 marks)

Q2 (a) Evaluate the following integrals using the tabular method.

(i) $\int e^{3x} \cos 3x \, dx$

(5 marks)

(ii) $\int_1^3 x^3 (x-3)^{\frac{5}{2}} dx$

(5 marks)

(b) Compute $\int \frac{x^2+1}{(x+2)^2}$ dx using partial fraction method.

(6 marks)

(c) Solve the following integral functions.

(i) $\int \frac{\sin^3\theta}{\cos^2\theta}\, d\theta$

(4 marks)

(ii) $\int (\sin x)^{\frac{1}{2}} \cos^3 x\, dx$

(5 marks)

Q3 (a) For each of functions below find $f^{-1}(x)$.

(i) $f(x) = \frac{1}{2x+1}$

(2 marks)

(ii) $f(x) = e^x + 1$

(2 marks)

(b) Find the derivative of inverse function for the following functions.

(i) $f(x) = x^3 + 1$

(4 marks)

(i) $f(x) = \cot x$

(5 marks)

(c) Given $y = \sqrt{x^2-1}\cos^{-1} x$, prove that,

$$(\sqrt{x^2-1})\frac{dy}{dx} - \frac{xy}{\sqrt{x^2-1}} = -\frac{(x^2-1)}{\sqrt{1-x^2}}$$

(6 marks)

(d) Evaluate the following integral of trigonometric function

$$\int \frac{1}{\sqrt{4+2x-x^2}}\, dx$$

(6 marks)

Q4 (a) Find the derivative of the following functions.

(i) $f(x) = e^{2x+1}\ cot^{-1} x$

(3 marks)

(ii) $f(x) = \frac{\sec x}{\ln x}$

(3 marks)

(b) Evaluate $\int \frac{x}{\sqrt{6+2x-x^2}}$ using trigonometric substitution method.

(7 marks)

(c) Find the differentiation of the function $y = \frac{\cosh^{-1}(3x)}{\text{sech}^{-1}x^2}$.

(7 marks)

(d) Evaluate $\int \frac{1}{x^2\sqrt{x^2-9}}$ using hyperbolic substitution

(5 marks)

-END OF QUESTIONS-

FINAL EXAMINATION

SEMESTER / SESSION : SEM I 2022/2023
COURSE NAME : CALCULUS
PROGRAMME CODE : BEJ/BEV
COURSE CODE : BEE 10103

FORMULAE

Indefinite Integrals

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1$$

$$\int \frac{1}{x}\, dx = \ln|x| + C$$

$$\int \cos x\ dx = \sin x + C$$

$$\int \sin x\ dx = -\cos x + C$$

$$\int \sec^2 x\ dx = \tan x + C$$

$$\int \csc^2 x\ dx = -\cot x + C$$

$$\int \sec x \tan x\ dx = \sec x + C$$

$$\int \csc x \cot x\ dx = -\csc x + C$$

$$\int e^x dx = e^x + C$$

$$\int \cosh x\ dx = \sinh x + C$$

$$\int \sinh x\ dx = \cosh x + C$$

$$\int \operatorname{sech}^2 x\ dx = \tanh x + C$$

$$\int \operatorname{csch}^2 x\ dx = -\coth x + C$$

$$\int \operatorname{sech} x \tanh x\ dx = -\operatorname{sech} x + C$$

$$\int \operatorname{csch} x \coth x\ dx = -\operatorname{csch} x + C$$

Integration Of Inverse Functions

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{\sqrt{a^2 - x^2}}\, dx = \cos^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{a^2 + x^2}\, dx = \frac{1}{a}\cot^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{|x|\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{|x|\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2 + a^2}}\, dx = \sinh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{1}{\sqrt{x^2 - a^2}}\, dx = \cosh^{-1}\left(\frac{x}{a}\right) + C$$

$$\int \frac{-1}{|x|\sqrt{a^2 - x^2}}\, dx = \frac{1}{a}\operatorname{sech}^{-1}\left|\frac{x}{a}\right| + C$$

$$\int \frac{-1}{|x|\sqrt{a^2 + x^2}}\, dx = \frac{1}{a}\operatorname{csch}^{-1}\left|\frac{x}{a}\right| + C$$

$$\int \frac{1}{a^2 - x^2}\, dx = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & |x| < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & |x| > a \end{cases}$$

FINAL EXAMINATION

SEMESTER / SESSION : SEM I 2022/2023
COURSE NAME : CALCULUS
PROGRAMME CODE : BEJ/BEV
COURSE CODE : BEE 10103

FORMULAE

TRIGONOMETRIC/ HYPERBOLIC SUBSTITUTION

Expression	*Trigonometry*	*Hyperbolic*
$\sqrt{x^2+k^2}$	$x = k\tan\theta$	$x = k\sinh\theta$
$\sqrt{x^2-k^2}$	$x = k\sec\theta$	$x = k\cosh\theta$
$\sqrt{k^2-x^2}$	$x = k\sin\theta$	$x = k\tanh\theta$

WEIERSTRASS SUBSTITUTION

$\tan\frac{1}{2}x = t$		$\tan x = t$	
$\sin x = \frac{2t}{1+t^2}$ $\tan x = \frac{2t}{1-t^2}$	$\cos x = \frac{1-t^2}{1+t^2}$ $dx = \frac{2dt}{1+t^2}$	$\sin 2x = \frac{2t}{1+t^2}$ $\tan 2x = \frac{2t}{1-t^2}$	$\cos 2x = \frac{1-t^2}{1+t^2}$ $dx = \frac{dt}{1+t^2}$

IDENTITIES OF TRIGONOMETRY AND HYPERBOLIC

Trigonometric Functions	*Hyperbolic Functions*
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \frac{e^x - e^{-x}}{2}$
$\sin 2x = 2\sin x\cos x$	$\cosh x = \frac{e^x + e^{-x}}{2}$
$\cos 2x = \cos^2 x - \sin^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$= 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x\cosh x$
$= 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$1 + \tan^2 x = \sec^2 x$	$= 2\cosh^2 x - 1$
$1 + \cot^2 x = \csc^2 x$	$= 1 + 2\sinh^2 x$
$\tan 2x = \frac{2\tan x}{1-\tan^2 x}$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\sin(x \pm y) = \sin x\cos y \pm \sin y\cos x$	$\tanh 2x = \frac{2\tanh x}{1+\tanh^2 x}$
$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$	$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$
$2\sin ax\cos bx = \sin(a+b)x + \sin(a-b)x$	$\sinh(x \pm y) = \sinh x\cosh y \pm \sinh y\cosh x$
$2\sin ax\sin bx = \cos(a-b)x - \cos(a+b)x$	$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$
$2\cos ax\cos bx = \cos(a-b)x + \cos(a+b)x$	