CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	ELECTROMAGNETIC FIELDS AND WAVES
COURSE CODE	:	BEJ 20303
PROGRAMME CODE	:	BEJ
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION		1. ANSWER ALL QUESTIONS (**PART A & PART B**) 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **FOURTEEN (14)** PAGES

PART A OBJECTIVES (40 marks)

Q1 The followings are two governing laws of electrostatic fields:

(2 marks)

(i) Gauss's Law
(ii) Faraday's Law
(iii) Coulomb's Law
(iv) Ampere's Law

a. (i) and (ii)
b. (iii) and (iv)
c. (i) and (iii)
d. (ii) and (iv)

Q2 Dielectric strength is maximum electric field that a dielectric could tolerate without electrical breakdown. Dielectric breakdown occurs when ....

(2 marks)

(i) the dielectric becomes conducting.
(ii) electric field in the dielectric pulls electrons completely out from the molecules.
(iii) molecules in the dielectric are too weak to hold the electric field average acceleration.
(iv) molecules are polarized.

a. (i), (ii), and (iv)
b. (i), (iii), and (iv)
c. (i), (ii), and (iii)
d. All the above

Q3 Electric field, ***E*** that exists in a region consisting of two different mediums which satisfy the interface separating the medium condition can be solved by using boundary condition. The boundary conditions at an interface separate the followings **EXCEPT**:

(2 marks)

a. Dielectric and dielectric
b. Dielectric and conductor
c. Conductor and free space
d. Conductor and conductor

Q4 A coaxial conductor as illustrated in **FIGURE Q4** has an inner and outer radius, a and b. Gauss's Law is applied to obtain the electric field, $\boldsymbol{E} = \frac{Q}{2\pi\varepsilon r L}\hat{r}$ and potential difference, $V = \frac{Q}{2\pi\varepsilon L} ln\frac{b}{a}$. Thus, the capacitance, C of the coaxial conductor can be obtained from the formula of ....

(2 marks)

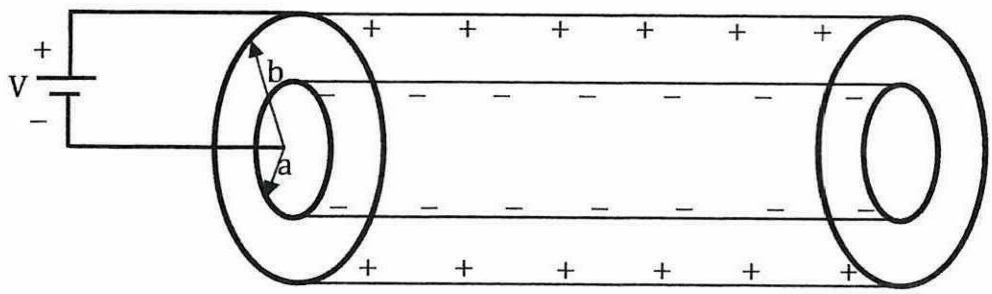

FIGURE Q4

a. $\frac{2\pi\varepsilon L}{\frac{\ln b}{a}}$

b. $\frac{ln\frac{b}{a}}{2\pi\varepsilon L}$

c. $2\pi\varepsilon L\left(\ln\frac{b}{a}\right)$

d. $\frac{V}{Q}$

Q5 External electric field $\boldsymbol{E}$ highly affects the electrons (negative charge) and protons (positive charge) of a dielectric material. When the direction of the $\boldsymbol{E}$ is along the positive y-axis, the protons are displaced to the __________ direction of $\boldsymbol{E}$, from their equilibrium position.

(2 marks)

a. opposite
b. same
c. negative
d. None of above

Q6. If the z-axis carries a filament current of 10π A along $\hat{z}$, which of the followings is **NOT TRUE**?

(2 marks)

a. $\boldsymbol{H} = -\hat{x}\ A/m$ at (0, 5, 0)
b. $\boldsymbol{H} = \hat{\phi}\ A/m$ at (5, π/4, 0)
c. $\boldsymbol{H} = -0.8\hat{x} - 0.6\hat{y}\ A/m$ at (-3, 4, 0)
d. $\boldsymbol{H} = -\hat{\phi}\ A/m$ at (5, 3π/2, 0)

Q7. Which of these statements is not the characteristic of a static magnetic field?

(2 marks)

a. It is a solenoidal.
b. It is conservative.
c. It has no sinks or sources.
d. Magnetic flux lines are always closed.

Q8. For the current and closed path of **FIGURE Q8**, calculate the line integral of $\oint_L \boldsymbol{H}\cdot d\boldsymbol{l}$.

(2 marks)

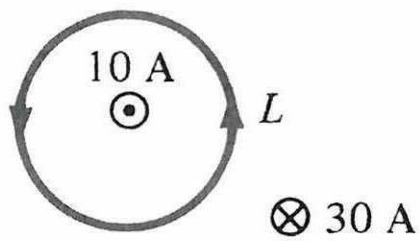

FIGURE Q8

a. 20 A
b. -20 A
c. 10 A
d. 30 A

Q9 The force exerted on differential length, d**l**, at point P along the conducting circular loop in **FIGURE Q9** is ……………

(2 marks)

a. Outward along OP
b. Inward along OP
c. In the direction of the magnetic field
d. Tangential to the loop at P

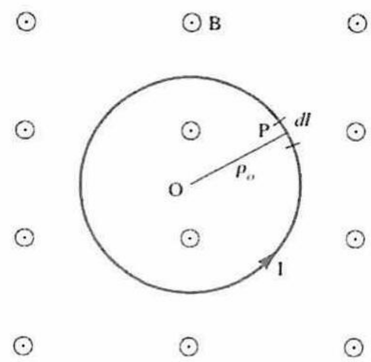

FIGURE Q9

Q10 Which of the following statements are **TRUE** about magnetic lines?

(2 marks)

(i) Magnetic lines start from the south pole and end at the north pole outside the magnet.
(ii) Magnetic lines are continuous.
(iii) They never intersect with each other.
(iv) Inside the magnet, the magnetic field lines are directed from south to north.

a. (i), (ii) and (iii)
b. (ii), (iii) and (iv)
c. (i), (iii) and (iv)
d. All of the above.

Q11 **FIGURE Q11** shows two identical bar magnets fixed in place with a proton moving to the right about to enter the region between two magnets. Which statement below best describe what happens to the proton while traveling between the magnets?

(2 marks)

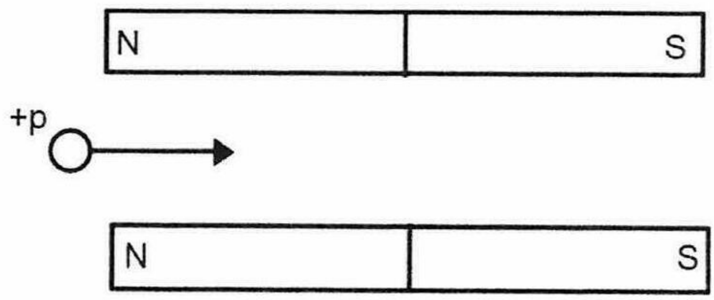

FIGURE Q11

a. It curves downward and strikes the bottom magnet

b. The proton continues to move to the right in a straight line

c. It curves upward and strikes the top magnet

d. The proton stops and moves to the left in a straight line

Q12 Which of the followings is the definition of Faraday's Law:

(2 marks)

a. It is a basic law of electromagnetism which demonstrates the interaction of magnetic flux and the induced voltage in a closed circuit.

b. It is defined as a relationship between current distribution and electric field intensity.

c. It is a combination of two different field components in normal and tangential directions respectively.

d. All of the above.

Q13 Mr. Faraday and Mr. Joseph Henry had conducted numerous experiments both in London and New York respectively and discovered the following:

(2 marks)

a. Electric current density is directly proportional to a conductive material's conductivity and electric field intensity.

b. Magnetic fields can induce an electric current in a closed loop, but only if the magnetic flux linking the surface of the loop changes with time.

c. A uniform magnetic field distribution can be generated on a filamentary wire

d. Electromotive force (EMF) can be induced in a closed loop based on the galvanometer's deflection.

Q14 If the number of turns in the coil is increased, the induced electromotive force in the coil will ____________________.

(2 marks)

a. Increase

b. Decrease

c. Remains same

d. None of the above

Q15 A __________________ of the galvanometer shows that the flow of induced__________________ is detected along the square loop. Choose the **CORRECT** answers to fill in the blank.

(2 marks)

a. momentry deflection, current
b. change in deflection, voltage
c. change in electric field, electromotive force
d. uniform change, motional electromotive force

Q16 Skin depth phenomenon is found in which material?

(2 marks)

a. Conductor
b. Insulator
c. Dielectric
d. Semiconductor

Q17 The electric field component of a wave in free space is given by $\boldsymbol{E} = 10cos(10^8 t - kz)\hat{y}$ V/m . It can be concluded that:

(2 marks)

(i) The wave propagates along -*z* direction
(ii) The wave number, *k* is 0.33 rad/m
(iii) The wavelength, λ is 18.85 m
(iv) The wave amplitude is 10 mV/m

a. (i) and (iv)
b. (ii) and (iii)
c. (i) and (iii)
d. (iii) and (iv)

Q18 Given that $\boldsymbol{H} = 0.5e^{-0.1x} sin(10^6 t - 2x)\hat{z}$ A/m, which of these statements is **NOT TRUE**?

(2 marks)

a. $\alpha = 0.1$ Np/m
b. $\beta = 2$ rad/m
c. $\omega = 10^6$ rad/s
d. The wave is polarized along z-direction.

Q19 What is the major factor in determining whether a medium is a lossless dielectric, lossy dielectric, or good conductor?

(2 marks)

a. Attenuation constant

b. Constitutive parameters (σ, ε, μ)

c. Loss tangent

d. Reflection coefficient

Q20 A blue beam of light that falls on two narrow slits produces an interference pattern on a screen. If a red beam of light is used in the same experiment instead of blue light, which changes can be observed in the interface pattern?

(2 marks)

a. Interference fringes move closer to the central maximum.

b. No changes in the interference pattern.

c. Interference fringes move further away from the maximum central.

d. Bright fringes are replaced with dark fringes.

PART B SUBJECTIVES (60 marks)

Q1 A spherical capacitor is formed by two metallic spheres of radius, a (inner sphere) and b (outer sphere), as shown in **Figure Q1**. The region between the two metallic spheres is filled with dielectric with a dielectric constant of ε_r. The charge on the inner sphere is $+Q$, and the outer sphere is $-Q$.

(a) State the law that can be used to find the total electric flux in the region between the spheres.

(1 mark)

(b) Based on your answer in Q1(a), sketch a diagram representing the use of the law in spherical capacitor applications.

(2 marks)

(c) By using the law stated in Q1(a), prove that the general formula to calculate electric field intensity in the region between the spheres is:

$$\boldsymbol{E} = \frac{Q}{4\pi\varepsilon r^2}\hat{r}$$

(5 marks)

(d) Find the general formula for the coaxial capacitor's electric potential, V.

(5 marks)

(e) Calculate the capacitance, C of the system.

(2 marks)

Q2 Two infinite current filaments are shown in **FIGURE Q2**.

(a) Calculate the magnetic field, $\boldsymbol{H}$ at (0, 0, 3) if the direction of both current filaments is along $\hat{z}$, $L1 = L2 = 3$ unit, $I_1 = 2$ A and $I_2 = 4$ A.

(6 marks)

(b) State two ways to determine the **ZERO** magnetic field at (0, 0, 3).

(2 marks)

(c) By ignoring the values given in **Q2(a)**, determine the force per unit length acting on I_2 if $I_1 = 6$ A and $I_2 = 8$ A, and $L_1 + L_2 = 10$ m with both currents flowing in the same direction.

(5 marks)

(d) If the current in I_1 and I_2 are in opposite directions, predict the direction of the force acting on I_2.

(2 marks)

Q3 (a) Faraday's Law states that the induced electromotive force (EMF), $V_{emf,}$ in any closed circuits is equal to the rate of change of time of the magnetic flux linkage by the circuit. With an aid of a diagram, discuss the experimental setup used to verify the Faraday's Law.

(4 marks)

(b) A sliding bar as shown in **FIGURE Q3(b)** is located at $x = 10t + 4t^3$, and the separation between the two rails is 40 cm. If the magnetic flux density, $\boldsymbol{B} = 0.8x^2\hat{z}$ Tesla, compute the voltmeter reading at $t = 0.8$ s.

(11 marks)

Q4 Seawater plays a vital role in the study of submarine communications. If the constitutive parameters of the seawater are given as $\sigma = 4$ S/m, $\varepsilon_r = 80$, $\mu_r = 1$ and $f = 100$ MHz:

(a) Show that seawater is a good conductor;

(2 marks)

(b) Calculate the attenuation constant, α and phase constant, β;

(3 marks)

(c) Calculate the wave velocity, u;

(2 marks)

(d) Calculate the wavelength, λ;

(2 marks)

(e) Calculate the skin depth, δ; and

(2 marks)

(f) Calculate the intrinsic impedance, η.

(4 marks)

-END OF QUESTIONS-

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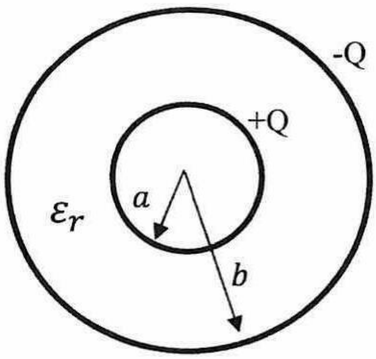

FIGURE Q1

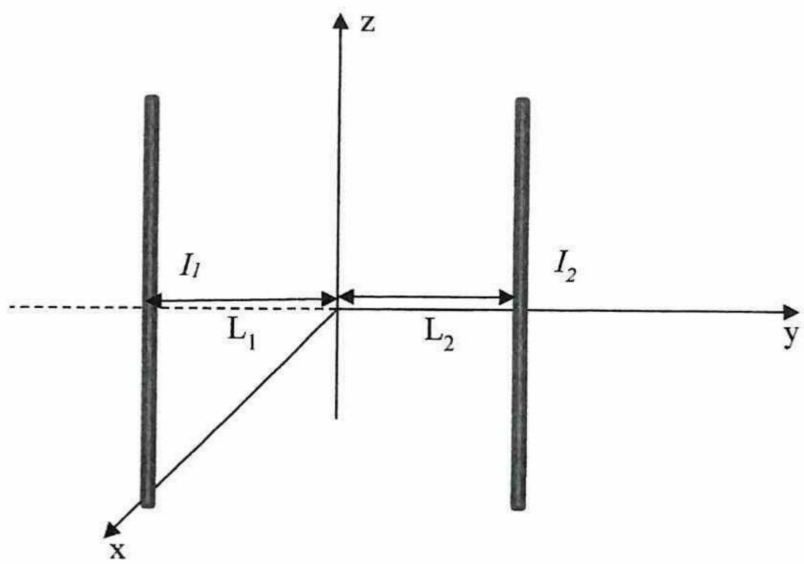

FIGURE Q2

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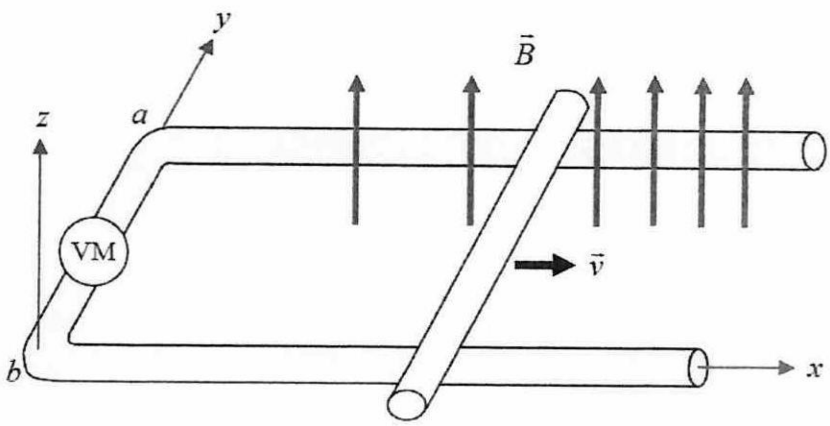

FIGURE Q3(b)

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Formula

	Cartesian	**Cylindrical**	**Spherical**
Coordinate parameters	x, y, z	r, ϕ, z	R, θ, ϕ
Vector $\vec{A}$	$A_x\hat{\mathbf{x}} + A_y\hat{\mathbf{y}} + A_z\hat{\mathbf{z}}$	$A_r\hat{\mathbf{r}} + A_\phi\hat{\boldsymbol{\varphi}} + A_z\hat{\mathbf{z}}$	$A_R\hat{\mathbf{R}} + A_\theta\hat{\boldsymbol{\theta}} + A_\phi\hat{\boldsymbol{\varphi}}$
Magnitude $\vec{A}$	$\sqrt{A_x^{\ 2} + A_y^{\ 2} + A_z^{\ 2}}$	$\sqrt{A_r^{\ 2} + A_\phi^{\ 2} + A_z^{\ 2}}$	$\sqrt{A_R^{\ 2} + A_\theta^{\ 2} + A_\phi^{\ 2}}$
Position vector, $\overrightarrow{OP}$	$x_1\hat{\mathbf{x}} + y_1\hat{\mathbf{y}} + z_1\hat{\mathbf{z}}$ for point $P(x_1,\ y_1,\ z_1)$	$r_1\hat{\mathbf{r}} + z_1\hat{\mathbf{z}}$ for point $P(r_1,\ \phi_1,\ z_1)$	$R_1\hat{\mathbf{R}}$ for point $P(R_1,\ \theta_1,\ \phi_1)$
Unit vector product	$\hat{\mathbf{x}}\bullet\hat{\mathbf{x}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{x}}\bullet\hat{\mathbf{y}} = \hat{\mathbf{y}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{x}} = 0$ $\hat{\mathbf{x}}\times\hat{\mathbf{y}} = \hat{\mathbf{z}}$ $\hat{\mathbf{y}}\times\hat{\mathbf{z}} = \hat{\mathbf{x}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{x}} = \hat{\mathbf{y}}$	$\hat{\mathbf{r}}\bullet\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{z}} = 1$ $\hat{\mathbf{r}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{z}} = \hat{\mathbf{z}}\bullet\hat{\mathbf{r}} = 0$ $\hat{\mathbf{r}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{z}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{z}} = \hat{\mathbf{r}}$ $\hat{\mathbf{z}}\times\hat{\mathbf{r}} = \hat{\boldsymbol{\varphi}}$	$\hat{\mathbf{R}}\bullet\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\boldsymbol{\varphi}} = 1$ $\hat{\mathbf{R}}\bullet\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\theta}}\bullet\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\bullet\hat{\mathbf{R}} = 0$ $\hat{\mathbf{R}}\times\hat{\boldsymbol{\theta}} = \hat{\boldsymbol{\varphi}}$ $\hat{\boldsymbol{\theta}}\times\hat{\boldsymbol{\varphi}} = \hat{\mathbf{R}}$ $\hat{\boldsymbol{\varphi}}\times\hat{\mathbf{R}} = \hat{\boldsymbol{\theta}}$
Dot product $\vec{A}\bullet\vec{B}$	$A_xB_x + A_yB_y + A_zB_z$	$A_rB_r + A_\phi B_\phi + A_zB_z$	$A_RB_R + A_\theta B_\theta + A_\phi B_\phi$
Cross product $\vec{A}\times\vec{B}$	$\begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\varphi}} & \hat{\mathbf{z}} \\ A_r & A_\phi & A_z \\ B_r & B_\phi & B_z \end{vmatrix}$	$\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} & \hat{\boldsymbol{\varphi}} \\ A_R & A_\theta & A_\phi \\ B_R & B_\theta & B_\phi \end{vmatrix}$
Differential length, $\overrightarrow{d\ell}$	$dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$	$dr\,\hat{\mathbf{r}} + rd\phi\,\hat{\boldsymbol{\varphi}} + dz\,\hat{\mathbf{z}}$	$dR\,\hat{\mathbf{R}} + Rd\theta\,\hat{\boldsymbol{\theta}} + R\sin\theta\,d\phi\hat{\boldsymbol{\varphi}}$
Differential surface, $\overrightarrow{ds}$	$\overrightarrow{ds}_x = dy\,dz\,\hat{\mathbf{x}}$ $\overrightarrow{ds}_y = dx\,dz\,\hat{\mathbf{y}}$ $\overrightarrow{ds}_z = dx\,dy\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_r = rd\phi\,dz\,\hat{\mathbf{r}}$ $\overrightarrow{ds}_\phi = dr\,dz\,\hat{\boldsymbol{\varphi}}$ $\overrightarrow{ds}_z = rdr\,d\phi\,\hat{\mathbf{z}}$	$\overrightarrow{ds}_R = R^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{R}}$ $\overrightarrow{ds}_\theta = R\sin\theta\,dR\,d\phi\,\hat{\boldsymbol{\theta}}$ $\overrightarrow{ds}_\phi = R\,dR\,d\theta\,\hat{\boldsymbol{\varphi}}$
Differential volume, $\overrightarrow{dv}$	$dx\,dy\ \ dz$	$r\,dr\,d\phi\,dz$	$R^2\sin\theta\,dR\,d\theta\,d\phi$

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$$Q = \int \rho_\ell d\ell,$$
$$Q = \int \rho_s dS,$$
$$Q = \int \rho_v dv$$
$$\overline{F}_{12} = \frac{Q_1 Q_2}{4\pi\varepsilon_0 R^2}\hat{a}_{R_{12}}$$
$$\overline{E} = \frac{\overline{F}}{Q},$$
$$\overline{E} = \frac{Q}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_s dS}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{E} = \int \frac{\rho_v dv}{4\pi\varepsilon_0 R^2}\hat{a}_R$$
$$\overline{D} = \varepsilon\overline{E}$$
$$\psi_e = \int \overline{D} \bullet d\overline{S}$$
$$Q_{enc} = \oint_S \overline{D} \bullet d\overline{S}$$
$$\rho_v = \nabla \bullet \overline{D}$$
$$V_{AB} = -\int_A^B \overline{E} \bullet d\overline{\ell} = \frac{W}{Q}$$
$$V = \frac{Q}{4\pi\varepsilon r}$$
$$V = \int \frac{\rho_\ell d\ell}{4\pi\varepsilon r}$$
$$\oint \overline{E} \bullet d\overline{\ell} = 0$$
$$\nabla \times \overline{E} = 0$$
$$\overline{E} = -\nabla V$$
$$\nabla^2 V = 0$$
$$R = \frac{\ell}{\sigma S}$$
$$I = \int \overline{J} \bullet dS$$

$$d\overline{H} = \frac{Id\overline{\ell} \times \overline{R}}{4\pi R^3}$$
$$Id\overline{\ell} \equiv \overline{J}_s dS \equiv \overline{J}dv$$
$$\oint \overline{H} \bullet d\overline{\ell} = I_{enc} = \int \overline{J}_s dS$$
$$\nabla \times \overline{H} = \overline{J}$$
$$\psi_m = \int_s \overline{B} \bullet d\overline{S}$$
$$\psi_m = \oint \overline{B} \bullet d\overline{S} = 0$$
$$\psi_m = \oint \overline{A} \bullet d\overline{\ell}$$
$$\nabla \bullet \overline{B} = 0$$
$$\overline{B} = \mu\overline{H}$$
$$\overline{B} = \nabla \times \overline{A}$$
$$\overline{A} = \int \frac{\mu_0 Id\overline{\ell}}{4\pi R}$$
$$\nabla^2 \overline{A} = -\mu_0 \overline{J}$$
$$\overline{F} = Q\left(\overline{E} + \overline{u} \times \overline{B}\right) = m\frac{d\overline{u}}{dt}$$
$$d\overline{F} = Id\overline{\ell} \times \overline{B}$$
$$\overline{T} = \overline{r} \times \overline{F} = \overline{m} \times \overline{B}$$
$$\overline{m} = IS\hat{a}_n$$
$$V_{emf} = -\frac{\partial \psi}{\partial t}$$
$$V_{emf} = -\int \frac{\partial \overline{B}}{\partial t} \bullet d\overline{S}$$
$$V_{emf} = \int \left(\overline{u} \times \overline{B}\right) \bullet d\overline{\ell}$$
$$I_d = \int J_d . d\overline{S}, J_d = \frac{\partial \overline{D}}{\partial t}$$
$$\gamma = \alpha + j\beta$$
$$\alpha = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} - 1\right]}$$
$$\beta = \omega\sqrt{\frac{\mu\varepsilon}{2}\left[\sqrt{1 + \left[\frac{\sigma}{\omega\varepsilon}\right]^2} + 1\right]}$$

$$\overline{F}_1 = \frac{\mu I_1 I_2}{4\pi}\oint_{L1}\oint_{L2}\frac{d\overline{\ell}_1 \times \left(d\overline{\ell}_2 \times \hat{a}_{R_{21}}\right)}{R_{21}^{\ 2}}$$
$$|\eta| = \frac{\sqrt{\mu/\varepsilon}}{\left[1 + \left(\frac{\sigma}{\omega\varepsilon}\right)^2\right]^{\frac{1}{4}}}$$
$$\tan 2\theta_\eta = \frac{\sigma}{\omega\varepsilon}$$
$$\tan\theta = \frac{\sigma}{\omega\varepsilon} = \frac{\overline{J}_s}{\overline{J}_{ds}}$$
$$\delta = \frac{1}{\alpha}$$
$$\varepsilon_0 = 8.854 \times 10^{-12} \ \text{Fm}^{-1}$$
$$\mu_0 = 4\pi \times 10^{-7} \ \text{Hm}^{-1}$$

$$\int \frac{dx}{\left(x^2 + c^2\right)^{3/2}} = \frac{x}{c^2\left(x^2 + c^2\right)^{1/2}}$$
$$\int \frac{xdx}{\left(x^2 + c^2\right)^{3/2}} = \frac{-1}{\left(x^2 + c^2\right)^{1/2}}$$
$$\int \frac{dx}{\left(x^2 \pm c^2\right)^{1/2}} = ln\left(x + \sqrt{x^2 \pm c^2}\right)$$
$$\int \frac{dx}{\left(x^2 + c^2\right)} = \frac{1}{c}tan^{-1}\left(\frac{x}{c}\right)$$
$$\int \frac{xdx}{\left(x^2 + c^2\right)} = \frac{1}{2}ln\left(x^2 + c^2\right)$$
$$\int \frac{xdx}{\left(x^2 + c^2\right)^{1/2}} = \sqrt{x^2 + c^2}$$