UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	ORDINARY DIFFERENTIAL EQUATIONS
COURSE CODE	:	BEE 11203
PROGRAMME CODE	:	BEJ / BEV
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS 2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

Q1 (a) Find a particular solution for the first order linear differential equation

$$e^x \frac{dy}{dx} + 2e^x y = 3,$$

given that $y = 0$ when $x = 2$.

(10 marks)

(b) Given

$$(4y + x^2)dy = (-2xy)dx$$

(i) Prove the above first order ordinary differential equation is non-homogenous. Show your calculation.

(4 marks)

(ii) Solve the above first order exact ordinary differential equation with initial condition of $y(0) = 1$.

(11 marks)

Q2 A second order non-homogeneous ordinary differential equation is given as

(a) $\frac{d^2y}{dx^2} + 4y = 3\sin(2x)$. Find the general solution using the method of undetermined coefficient.

(12 marks)

(b) $2y'' - 5y' + 2y = 7xe^{2x}$. Find the general solution using the method of variation of parameters.

(13 marks)

Q3

$$y_1{}' = 5y_1 - 2y_2 + 2e^{3x}$$
$$y_2{}' = 3y_1 - 2y_2 + 10x - 5$$

is a system of first-order linear differential equations.

(a) Calculate the complementary function, $\boldsymbol{Y_C}$, for the homogeneous system.

(10 marks)

(b) Calculate the particular integral, $\boldsymbol{Y_P}$, for the non-homogeneous system.

(10 marks)

(c) Determine the general solution, $\boldsymbol{Y}$, for the non-homogeneous system.

(1 mark)

(d) Obtain the particular solution for $y_1(x)$ and $y_2(x)$ with $\boldsymbol{Y}(0) = \binom{1}{1}$.

(4 marks)

Q4 (a) Solve;

(i) the Laplace transform of function $y(t)$ in **Figure Q4(a)**

(4 marks)

(ii) the inverse Laplace transform of

$$F(s) = \frac{s^2 + 5}{s(s^2 + 10)}$$

(6 marks)

(b) The RLC circuit which is initially at rest, is modeled as

$$L\frac{di(t)}{dt} + i(t)R + \frac{1}{C}\int_0^t i(\tau)\,d\tau = \begin{cases} 1, & t < 10 \\ 0, & t \geq 10 \end{cases}$$

with $L = 1\ H$, $R = 0.4\ \Omega$, and $C = 8\ F$.

Determine the current $i(t)$ using Laplace transform.

(15 marks)

-END OF QUESTIONS –

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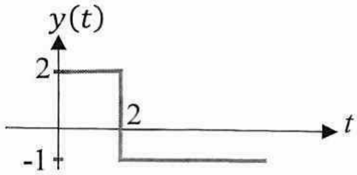

Figure Q4(a)

Table 1: Second-Order Differential Equation

The roots of characteristic equation and the general solution for differential equation
$ay''(t) + by'(t) + cy(t) = 0.$

Characteristic equation: $am^2 + bm + c = 0.$		
Case	The roots of characteristic equation	General solution
1.	Real and different roots: m_1 and m_2	$y = Ae^{m_1 t} + Be^{m_2 t}$
2.	Real and equal roots: $m = m_1 = m_2$	$y = (A + Bt)e^{mt}$
3.	Complex roots: $m_1 = \alpha + \beta i, m_2 = \alpha - \beta i$	$y = e^{\alpha t}(A \cos \beta\, t + B \sin \beta\, t)$

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Table 2: Particular Integral, Y_p based on $f(x)$

Type of $f(x)$	Example of $f(x)$	Assumption of y_p
Exponent	ke^{nx}	Ce^{nx}
Polynomial	k	C
	kx	$Cx+D$
	kx^2	Cx^2+Dx+E
	kx^n	$C_nx^n+C_{n-1}x^{n-1}+\ldots+C_1x+C_0$
Trigonometry (sin and cos only)	$k\sin nx$ or $k\cos nx$	$y_p=C\cos nx+D\sin nx$
	$k\sinh nx$ or $k\cosh nx$	$y_p=C\cosh nx+D\sinh nx$
Product of polynomial and exponential	$P_n(x)e^{nx}$	$(C_nx^n+C_{n-1}x^{n-1}+\ldots+C_1x+C_0)e^{nx}$
Product of polynomial and trigonometry	$P_n(x)\sin nx$ or $P_n(x)\cos nx$	$(C_nx^n+C_{n-1}x^{n-1}+\ldots+C_1x+C_0)\sin nx+$ $(D_nx^n+D_{n-1}x^{n-1}+\ldots+D_1x+D_0)\cos nx$
Product of exponential and trigonometry	$ke^{nx}\sin nx$	$e^{nx}\left(C\cos nx+D\sin nx\right)$

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Table 3: Particular Integral, Y_p based on G

Assume $\mathbf{Y}_p$ based on $\mathbf{G}$		
Case	$\mathbf{G}$	$\mathbf{Y}_p$
Case I	Polynomial $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix};\begin{pmatrix} a_1x+b_1 \\ a_2x+b_2 \end{pmatrix};$ $\begin{pmatrix} a_1x^2+b_1x+c_1 \\ a_2x^2+b_2x+c_2 \end{pmatrix}$	$\begin{pmatrix} C \\ D \end{pmatrix};\begin{pmatrix} Cx+E \\ Dx+F \end{pmatrix};$ $\begin{pmatrix} Cx^2+Ex+G \\ Dx^2+Fx+H \end{pmatrix}$
Case II	Exponent $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}e^{kx}$	$\begin{pmatrix} C \\ D \end{pmatrix}e^{kx}$ if $\mathbf{Y}_p \equiv \mathbf{Y}_C$, then $\begin{pmatrix} C \\ D \end{pmatrix}xe^{kx}+\begin{pmatrix} E \\ F \end{pmatrix}e^{kx}$
Case III	Trigonometric (sin and cos only) $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\sin kx$ or $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix}\cos kx$	$\begin{pmatrix} C \\ D \end{pmatrix}\sin kx+\begin{pmatrix} E \\ F \end{pmatrix}\cos kx$

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Table 4: Laplace Transform

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$\int_0^t f(\tau)\,d\tau$	$\frac{F(s)}{s}$
e^{at}	$\frac{1}{s-a}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\sinh at$	$\frac{a}{s^2-a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\int_0^t f(u)g(t-u)\,du$	$F(s)\cdot G(s)$
t^n, $n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$y'(t)$	$sY(s)-y(0)$
$t^n f(t)$, $n=1,2,3,\ldots$	$(-1)^n \frac{d^n}{ds^n}F(s)$	$y''(t)$	$s^2Y(s)-sy(0)-y'(0)$

Table 5: Electrical Formula

No.	Item	Formula
1	Voltage drop across resistor, R (Ohm's Law)	$v_R = iR$
2	Voltage drop across inductor, L (Faraday's Law)	$v_L = L\frac{di}{dt}$
3	Voltage drop across capacitor, C (Coulomb's Law)	$v_c = \frac{q}{C}$ or $i = C\frac{dv_c}{dt}$
4	The relation between current, i and charge, q	$i = \frac{dq}{dt}$