Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	CALCULUS / CIVIL ENGINEERING MATHEMATICS 1
COURSE CODE	:	BFC 15003 / BFC 13903
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER **ALL** QUESTIONS 2. THIS FINAL EXAMINATION CONDUCTED VIA **CLOSED BOOK** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

Q1 (a) Determine the derivative of $f(t) = \frac{4}{3}\tan(3t^2 - 4\pi)$.

(4 marks)

(b) An equation is given as $2x^2y = 4x - e^{2x}y^2$. Determine the first derivative of the equation in terms of x.

(6 marks)

(c) By using a suitable method, determine the first and second derivatives of the following function;

$$g(x) = \frac{2x^2 - 2}{4x^2 + 2x - 4}$$

(7 marks)

(d) A steel bar with a radius r as illustrated in **Figure Q1(d)**, is continuously heated for approximately 2 hours and 30 minutes to observe how heat affects the bar's tendency to elongate or expand. It has been observed that, the radius r of the steel bar increases at a steady rate of 0.2mm/hour.

(i) State the total surface area of the steel bar, A in terms of its radius r and find $\frac{dA}{dr}$.

(3 marks)

(ii) If the initial length l and radius r of the steel bar are 500 mm and 10 mm respectively. Examine the length l and radius r of the steel bar after being subjected to heat for 45 minutes.

(4 marks)

(iii) Based on **Q1(d)(i) and (ii)** analyse the rate of surface area expansion of the bar at that time.

(4 marks)

Q2 (a) Find the gradient of a curve $y = 2x^3 + 16x - 4$ at (0, -4) and (2, 5).

(2 marks)

(b) Determine intervals from the following equation and state either it is increasing or decreasing.

$$y = x^{3-}9x^2 + 24x$$

(8 marks)

(c) Solve the following definite integral with respect to x.

$$\int_{1}^{2} 3x^2 - 5x + 2\, dx$$

(8 marks)

Q3 (a) Express the following indefinite integral in partial fractions.

$$\int \frac{x-4}{x^2+2x-15}\, dx$$

(10 marks)

(b) Determine the area bounded by the two functions $f(x) = cos\ x$ and $g(x) = sin\ x$ on the interval $(\frac{\pi}{4}, \frac{5\pi}{4},)$.

(9 marks)

(c) The region R is bounded by the curve $y = xe^{-1/2x}$, the line $x = 4$ and the x-axis.

(i) Show that the area of the regions is $\frac{4(e^2-3)}{e^2}$ units^2.

(5 marks)

(ii) The region R is rotated through 2π radians about the x-axis. Determine the volume of the solid generated.

(8 marks)

Q4 (a) The gradient of a curve at point (x, y) satisfies the differential equation $\frac{dy}{dx} - 9xy\sqrt{3x^2-2}$. Obtain the general solution of the differential equation and find the equation of the curve that passes through point $(1, -e)$.

(7 marks)

(b) An object moves along a straight line and passes a fixed-point O with velocity u in the positive direction of the x-axis. At time t, the object is at a displacement x from O and the velocity of the object is v. the rate of change of velocity has magnitude $\frac{k}{v^2}$, where k is a constant and is directed to towards the fixed-point O.

(ii) Write down a differential equation for the motion of the object involving the velocity v and the time t. hence, find the velocity v as a function of the time t.

(6 marks)

(ii) Show that $\frac{dv}{dt} = v\frac{dv}{dx}$ and hence, write down the differential equation for the motion of the object. Hence find the velocity v as a function of the displacement x. Hence, show that after a time t and the object has moved a distance $x, 4kx = u^4 - (u^3 - 3kt)^{\frac{4}{3}}$.

(9 marks)

- END OF QUESTIONS -

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Figure Q1(d) – Steel Bar

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Formula

Differentiation Rules	Indefinite Integrals
$\frac{d}{dx}[k]=0$	$\int k\,dx = kx + C$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\int x^n\,dx = \frac{x^{n-1}}{n+1}+C \qquad n\neq -1$
$\frac{d}{dx}[\ln\|x\|]=\frac{1}{x}$	$\int \frac{dx}{x} = \ln\|x\| + C$
$\frac{d}{dx}[\cos x]=-\sin x$	$\int \sin x\,dx = -\cos x + C$
$\frac{d}{dx}[\sin x]=\cos x$	$\int \cos x\,dx = \sin x + C$
$\frac{d}{dx}[\tan x]=\sec^2 x$	$\int \sec^2 x\,dx = \tan x + C$
$\frac{d}{dx}[\cot x]=-\text{cosec}^2 x$	$\int \text{cosec}^2 x\,dx = -\cot x + C$
$\frac{d}{dx}[\sec x]=\sec x\tan x$	$\int \sec x\tan x\,dx = \sec x + C$
$\frac{d}{dx}[\text{cosec}\, x]=-\text{cosec}\, x\cot x$	$\int \text{cosec}\, x\cot x\,dx = -\text{cosec}\, x + C$
$\frac{d}{dx}[e^x]=e^x$	$\int e^x\,dx = e^x + C$
$\frac{d}{dx}[\cosh x]=\sinh x$	$\int \sinh x\,dx = \cosh x + C$
$\frac{d}{dx}[\sinh x]=\cosh x$	$\int \cosh x\,dx = \sinh x + C$
$\frac{d}{dx}[\tanh x]=\text{sech}^2 x$	$\int \text{sech}^2 x\,dx = \tanh x + C$
$\frac{d}{dx}[\coth x]=-\text{cosech}^2 x$	$\int \text{cosech}^2 x\,dx = -\coth x + C$
$\frac{d}{dx}[\text{sech}\, x]=-\text{sech}\, x\tanh x$	$\int \text{sech}\, x\tanh x\,dx = -\text{sech}\, x + C$
$\frac{d}{dx}[\text{cosech}\, x]=-\text{cosech}\, x\coth x$	$\int \text{cosech}\, x\coth x\,dx = -\text{cosech}\, x + C$

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Trigonometric	Hyperbolic
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \csc^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \operatorname{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \operatorname{csch}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x + y) + \sin(x - y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x + y) + \cos(x - y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x + y) + \cos(x - y)$	
	$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$, any x
$a^x = e^{x \ln a}$	$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$, $x \geq 1$
$\log_a x = \dfrac{\log_b x}{\log_b a}$	$\tanh^{-1} x = \dfrac{1}{2}\ln\left(\dfrac{1 + x}{1 - x}\right)$, $-1 < x < 1$

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Integration of Inverse Function
$\int \frac{dx}{\sqrt{a^2 - x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{a^2 + x^2} = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{\lvert a\rvert\sqrt{a^2 - x^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$
$\int \frac{dx}{\sqrt{x^2 + a^2}} = \sinh^{-1}\left(\frac{x}{a}\right) + C, \quad a > 0$
$\int \frac{dx}{\sqrt{x^2 - a^2}} = \cosh^{-1}\left(\frac{x}{a}\right) + C, \quad x > a$
$\int \frac{dx}{x^2 - a^2} = \begin{cases} \frac{1}{a}\tanh^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert < a \\ \frac{1}{a}\coth^{-1}\left(\frac{x}{a}\right) + C, & \lvert x\rvert > a \end{cases}$
$\int \frac{dx}{x\sqrt{a^2 - x^2}} = -\frac{1}{a}\operatorname{sech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a$
$\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a}\operatorname{cosech}^{-1}\left(\frac{x}{a}\right) + C, \quad 0 < x < a$

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Formula

Differentiation of Inverse Functions	
y	$\frac{dy}{dx}$
$\sin^{-1} u$	$\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad \lvert u\rvert<1$
$\cos^{-1} u$	$-\frac{1}{\sqrt{1-u^2}}\frac{du}{dx}, \quad \lvert u\rvert<1$
$\tan^{-1} u$	$\frac{1}{1+u^2}\frac{du}{dx}$
$\cot^{-1} u$	$-\frac{1}{1+u^2}\frac{du}{dx}$
$\sec^{-1} u$	$\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert>1$
$\operatorname{cosec}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert>1$
$\sinh^{-1} u$	$\frac{1}{\sqrt{u^2+1}}\frac{du}{dx}$
$\cosh^{-1} u$	$\frac{1}{\sqrt{u^2-1}}\frac{du}{dx}, \quad \lvert u\rvert>1$
$\tanh^{-1} u$	$\frac{1}{1-u^2}\frac{du}{dx}, \quad \lvert u\rvert<1$
$\coth^{-1} u$	$-\frac{1}{1-u^2}\frac{du}{dx}, \quad \lvert u\rvert>1$
$\operatorname{sech}^{-1} u$	$-\frac{1}{u\sqrt{1-u^2}}\frac{du}{dx}, \quad 0<u<1$
$\operatorname{cosech}^{-1} u$	$-\frac{1}{\lvert u\rvert\sqrt{1+u^2}}\frac{du}{dx}, \quad u\neq 0$