CONFIDENTIAL

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2022/2023

| | | |
|---|---|---|
| COURSE NAME | : | CIVIL ENGINEERING MATHEMATICS III |
| COURSE CODE | : | BFC 24103 |
| PROGRAMME CODE | : | BFF |
| EXAMINATION DATE | : | FEBRUARY 2023 |
| DURATION | : | 3 HOURS |
| INSTRUCTION | : | 1. ANSWER **ALL** QUESTIONS<br>2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.**<br>3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK. |



THIS QUESTION PAPER CONSISTS OF **SIX (6)** PAGES

**Q1** (a) The position vector of a particle is

$$\mathbf{r}(t) = \cos 2t\boldsymbol{i} + 2\sin 2t\,\boldsymbol{j} + \boldsymbol{t^2k}$$

Find the velocity, speed, direction and acceleration of the particle at $t = \pi$

(8 marks)

(b) Calculate the unit tangent vector, unit normal vector and curvature of the given vector-valued function.

$$\mathbf{r}(t) = t\mathbf{i} + 3\cos t\mathbf{j} + 3\sin t\mathbf{k}$$

(12 marks)

**Q2** (a) By using Stokes' theorem, evaluate $\oint_c \mathbf{F}.d\mathbf{r}$ for the vector $\mathbf{F} = xz\boldsymbol{i} + xy^2\boldsymbol{j} + 3xz\boldsymbol{k}$ and the space curve $C$ which is the intersection of the plane $x + z = 3$ and the cylinder $x^2 + y^2 = 4$, in the counterclockwise direction, when viewed from the positive z-axis.

(10 marks)

(b) By using Green's theorem, evaluate the following equation where $C$ is the boundary of the region in first quadrant bounded by $x^2 - 2x + y^2 = 0$ and $y = x$.

$$\oint_C (2Y - x^2)dx + (4x + y^2)dy$$

(10 marks)

**Q3** (a) Calculate the rate at which the pressure $P$ (kpa) is changing when the temperature $T$ (K) and increasing at rate of 0.1 K/s and volume $V$ (L) is 100 L and increasing rate of 0.2 L/s. Solve the problem by using following equation

$$PV = 8.31T$$

(10 marks)

(b) The dimensions of a bridge are change from 15 m, 13 m and 11 m to 15.03 m, 12.96 m and 11.02 m. Calculate the approximate change in volume by using total differential.

(4 marks)

(c) The radius of a right circular cylinder is measured with an error of at most 4% and the height is measured with and error of at most 8%. Approximate the maximum possible percentage error in the volume $V$ calculated from these measurements.

(6 marks)

**Q4** (a) The region 'R' is a triangle and it located on xy-plane. If the given planes is $4x + 2y + z = 4$, $x = 0$, $y = 0$ and $z = 0$, calculate the volume of the tetrahedron region bounded.

(7 marks)

(b) A lamina has shape of the region in the first quadrant that is bounded by the graphs of $y = \sin x$, $y = \cos x$, between $x = 0$ and $x = \pi/4$. Determine the centre of mass if the density is $y$.

(10 marks)

**Q5** (a) Given $\iiint\limits_E e^{-x^2-z^2}\, dV$ where $E$ is the region between the two cylinders $x^2 + z^2 = 4$ and $x^2 + z^2 = 9$ with $1 \leq y \leq 5$ and $z \leq 0$. Convert the given integral into cylindrical coordinates and analyse it.

(10 marks)

(b) Analyse the mass of the object which is bounded above by the inverted paraboloid $z = 16 - x^2 - y^2$ and below by the xy-plane. The density of the object is given by $\delta(x, y, z) = 8 + x + y$.

(10 marks)

**- END OF QUESTIONS –**

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**Formulae**

Tangent Plane: $z - z_0 = f_x(x_o, y_0)(x - x_0) + f_y(x_o, y_0)(y - y_0)$

Local Extreme Value: $G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - [f_{xy}(x,y)]^2$

| Case | $G(a,b)$ | Result |
|---|---|---|
| 1 | $G(a,b) > 0$ <br> $f_{xx}(a,b) < 0$ | $f(x,y)$ has a local maximum value at $(a,b)$ |
| 2 | $G(a,b) > 0$ <br> $f_{xx}(a,b) > 0$ | $f(x,y)$ has a local minimum value at $(a,b)$ |
| 3 | $G(a,b) < 0$ | $f(x,y) has\ a\ saddle\ point\ at\ (a,b)$ |
| 4 | $G(a,b) = 0$ | inconclusive |

Polar coordinate: $x = rcos\,\theta, y = rsin\,\theta, \theta = tan^{-1}(\frac{y}{x})$ and
$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical coordinate: $x = rcos\,\theta,$ , $y = rsin\,\theta,$ $z = z,$ $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)d\ dz\ dr\ d\theta$

Spherical coordinate: $x = \rho \sin\phi \cos\theta$, $y = \rho \sin\phi \sin\theta$, $z = \rho\cos\theta$, $x^2 + y^2 + z^2 = \rho^2$, $0 \ll \theta \ll 2\pi$, $0 \ll \phi \ll \pi$ and $\iiint f(x,y,z)dV = \iiint f(\rho,\phi,\theta)\rho^2 \sin\phi\ d\rho d\phi d\theta$

For lamina
Mass, $m = \iint_R \delta(x,y)\ dA$
Moment of mass: y-axis: $M_y = \iint_R x\delta(x,y)\ dA$ x-axis, $M_x = \iint_R y\delta(x,y)\ dA$
Center of mass, $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$
Centroid for homogenous lamina: $\bar{x} = \frac{1}{area}\iint_R x\ dA$ $\bar{y} = \frac{1}{area}\iint_R y\ dA$
Moment inertia:
Y-axis: $I_y = \iint_R x^2\,\delta(x,y)\ dA$ x-axis: $I_x = \iint_R y^2\,\delta(x,y)\ dA$
Z-axis (or origin): $I_z = I_0 = \iint_R (x^2 + y^2)\,\delta(x,y)\ dA = I_x + I_y$

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For solid

Mass, $m = \iiint_G \delta(x,y)\, dV$

Moment of mass:

yz-plane: $M_{yz} = \iiint_G x\,\delta(x,y,z)\, dV$

xz-plane: $M_{xz} = \iiint_G y\,\delta(x,y,z)\, dV$

xy-plane: $M_{xy} = \iiint_G z\,\delta(x,y,z)\, dV$

Center of gravity, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m})$

Moment inertia:

$$I_y = \iiint_G (x^2 + z^2)\,\delta(x,y,z)\, dV$$

$$I_x = \iiint_G (y^2 + z^2)\,\delta(x,y,z)\, dV$$

$$I_z = \iiint_G (x^2 + y^2)\,\delta(x,y,z)\, dV$$

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Directional derivative: $D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot u$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then the divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The curl of

$$\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let $C$ is a smooth curve given by $r(t) = x(t)i + y(t)j + z(t)k$, $t$ is parameter, then

The unit tangent vector; $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The binormal vector: $B(t) = T(t) \times N(t)$

The curvature: $K = \frac{\mathbf{T}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}' \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

The radius of curvature: $\rho = 1/K$

Green Theorem: $\oint_C M(x,y)\, dx + N(x,y)\, dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} dA$

Gauss Theorem: $\iint_\sigma \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_G \nabla.\mathbf{F}\, dV$

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Stokes Theorem: $\oint_C \mathbf{F}.d\mathbf{r} = \iint_\sigma (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$

Arc length, If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\, dt = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$$

If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2}\, dt$$