UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	ENGINEERING MATHEMATICS
COURSE CODE	:	BFC 25103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1.ANSWER ALL QUESTIONS 2.THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 3.STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) Assuming that $L\{e^{at}\} = \frac{1}{s-a}$ using the formula $L\{t^n f(t)\} = (-1)^n \frac{d^n}{s^n}\ [F(s)]$ find the following functions

i) $L\{te^{at}\}$

(4 marks)

ii) $L\{t^2 e^{at}\}$

(3 marks)

(b) Calculate the Laplace transform by Applying the theorem of the first shift for the function.

$$L = \{e^{-2t} cos\ 3t\}$$

(3 marks)

(c) Using partial fractions, Determine the Inverse Laplace transforms for the equation.

$$L^{-1}\left\{\frac{3s^3 + s^2 + 12s + 2}{(s-3)(s+1)^3}\right\}$$

(7 marks)

(d) Assume an object weighing 2 lb stretches a spring 6 in. Establish the equation of motion if the spring is released from the equilibrium position with an upward velocity of 16 ft/sec. Discover the period of the motion.

(8 marks)

Q2 (a) Classify the given function as continuous or not continuous

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x + y}, & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$$

(6 marks)

(b) Calculate the partial differentiation for each of the following functions

i) $z = xy\ cos(xy)$

(3 marks)

ii) $z = (3x^2 + y)^2$

(3 marks)

(c) Using partial derivatives estimate the maximum possible error in calculating surface area of closed rectangular box with dimension of 3 m, 4 m, and 5 m respectively and the possible error 1/192 m.

(6 marks)

(d) Based on the answer of the Q2 (c) and the finding of possible percentage error. Determine the minimum possible percentage error up to the highest percentage.

(7 marks)

Q3 (a) Solve the $\iint_R e^{x2} \; dA$ where R is the region between x − axis, lines $y = \frac{x}{2}$ and x = 4 by using double integrals equations.

(5 marks)

(b) By applying polar coordinates, calculate and sketch a diagram of the region $\iint_R x + y \, dxdy$ over a region bounded by curves $xy = 6 \; and \; x + y = 7,$

(8 marks)

(c) Determine the centre of mass of the triangle with boundaries $y = 0$, $x = 1$ and $y = 2x$, and mass density $\rho(x, y) = x + y$.

(6 marks)

(d) The first octant bounded by $0 \leq x, 3x \leq y, 0 \leq z \; and \; y^2 + z^2 \leq 9$. Solve the triple integral of $f(x, y, z) = z$

(6 marks)

Q4 (a) Find the length of the arc of the circular helix from the point (1, 0, 0) to the point (1, 0, 6), If $r(t) = 3cos\, t\, i + 3sin\, t\, j + 3t\, k$.

(6 marks)

(b) Curvature of a circle of radius a is $1/a$, based on your understanding, explain why.

(6 marks)

(c) Given $r(t) = 3cos\, t\, i + 3sin\, t\, j + 3t\, k$ calculate the unit normal and binomial vectors for the circular helix

(7 marks)

(d) Make use of the force field $\mathbf{F}(x, y, z) = (z^3 - 2xy)\mathbf{i} - x^2\mathbf{j} + 3xz^2\mathbf{k}$, and show that **F** is a conservative vector field.

(6 marks)

-END OF QUESTIONS-

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{ex}(A\cos\beta\, x + B\sin\beta\, x)$

Laplace Transforms

$$\mathbf{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n , $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}
$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s) - y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s) - sy(0) - \dot{y}(0)$
$t^n f(t)$, $n = 1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$		

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Periodic Function for Laplace transform : $\mathrm{L}\{f(t)\} = \frac{1}{1-e^{-sT}} \int_0^T e^{-st} f(t)\,dt, \quad s > 0.$

Table .1 Inverse Laplace transforms

$F(s) = \mathcal{L}[f(t)]$	$\mathcal{L}^{-1}[F(s)] = f(t)$
(i) $\frac{1}{s}$	1
(ii) $\frac{k}{s}$	k
(iii) $\frac{1}{s-a}$	e^{at}
(iv) $\frac{a}{s^2+a^2}$	$\sin at$
(v) $\frac{s}{s^2+a^2}$	$\cos at$
(vi) $\frac{1}{s^2}$	t
(vii) $\frac{2!}{s^3}$	t^2
(viii) $\frac{n!}{s^{n+1}}$	t^n
(ix) $\frac{a}{s^2-a^2}$	$\sinh at$
(x) $\frac{s}{s^2-a^2}$	$\cosh at$
(xi) $\frac{n!}{(s-a)^{n+1}}$	$e^{at}t^n$
(xii) $\frac{\omega}{(s-a)^2+\omega^2}$	$e^{at}\sin \omega t$
(xiii) $\frac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos \omega t$
(xiv) $\frac{\omega}{(s-a)^2-\omega^2}$	$e^{at}\sinh \omega t$
(xv) $\frac{s-a}{(s-a)^2-\omega^2}$	$e^{at}\cosh \omega t$

Case	$\boldsymbol{G(a,b)}$	**Result**
1	$G(a,b) > 0$ $f_{xx}(a,b) < 0$	$f(x,y)$ has a local maximum value at (a,b)
2	$G(a,b) > 0$ $f_{xx}(a,b) > 0$	$f(x,y)$ has a local minimum value at (a,b)
3	$G(a,b) < 0$	$f(x,y) \, has\ a\ saddle\ point\ at\ (a,b)$
4	$G(a,b) = 0$	inconclusive

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Formulae

Tangent Plane: $z - z_0 = f_x(x_o, y_0)(x - x_0) + f_y(x_o, y_0)(y - y_0)$

Local Extreme Value: $G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - [f_{xy}(x,y)]^2$

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = tan^{-1}(\frac{y}{x})$ and
$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical coordinate: $x = r\cos\theta$, , $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)d\, dz\, dr\, d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\theta$, $x^2 + y^2 + z^2 = \rho^2$, $0 \ll \theta \ll 2\pi$, $0 \ll \phi \ll \pi$ and $\iiint f(x,y,z)dV = \iiint f(\rho,\phi,\theta)\rho^2\sin\phi\, d\rho d\phi d\theta$

For lamina
Mass, $m = \iint_R \delta(x,y)\, dA$
Moment of mass: y-axis: $M_y = \iint_R x\delta(x,y)\, dA$ x-axis, $M_x = \iint_R y\delta(x,y)\, dA$
Center of mass, $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$
Centroid for homogenous lamina: $\bar{x} = \frac{1}{area}\iint_R x\, dA$ $\bar{y} = \frac{1}{area}\iint_R y\, dA$
Moment inertia:
Y-axis: $I_y = \iint_R x^2\,\delta(x,y)\, dA$ x-axis: $I_x = \iint_R y^2\,\delta(x,y)\, dA$
Z-axis (or origin): $I_z = I_0 = \iint_R (x^2 + y^2)\,\delta(x,y)\, dA = I_x + I_y$

For solid
Mass, $m = \iiint_G \delta(x,y)\, dV$
Moment of mass:
yz-plane: $M_{yz} = \iiint_G x\,\delta(x,y,z)\, dV$
xz-plane: $M_{xz} = \iiint_G y\,\delta(x,y,z)\, dV$
xy-plane: $M_{xy} = \iiint_G z\,\delta(x,y,z)\, dV$
Center of gravity, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m})$

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Moment inertia:

$$I_y = \iiint_G (x^2 + z^2)\, \delta(x, y, z)\, dV$$

$$I_x = \iiint_G (y^2 + z^2)\, \delta(x, y, z)\, dV$$

$$I_z = \iiint_G (x^2 + y^2)\, \delta(x, y, z)\, dV$$

Directional derivative: $D_u f(x, y) = \left(f_x\mathbf{i} + f_y\mathbf{j}\right) \cdot u$

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then the divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The curl of

$$\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let C is a smooth curve given by $r(t) = x(t)i + y(t)j + z(t)k, t$ is parameter, then
The unit tangent vector; $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$
The unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$
The binormal vector: $B(t) = T(t) \times N(t)$
The curvature: $K = \frac{\mathbf{T}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}' \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$
The radius of curvature: $\rho = 1/K$

Green Theorem: $\oint_C M(x, y)\, dx + N(x, y)\, dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} dA$
Gauss Theorem: $\iint_\sigma \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_G \nabla . \mathbf{F}\, dV$
Stokes Theorem: $\oint_C \mathbf{F}. d\mathbf{r} = \iint_\sigma (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$

Arc length, If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\epsilon[a, b]$, then the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\, dt = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2}\, dt$$

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If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \sqrt{\left(x'(t)\right)^2 + \left(y'(t)\right)^2 + \left(z'(t)\right)^2}\; dt$$

Trigonometric and Hyperbolic Identities

Trigonometric

$$\cos^2 x + \sin^2 x = 1$$
$$1 + \tan^2 x = \sec^2 x$$
$$\cot^2 x + 1 = \operatorname{cosec}^2 x$$
$$\sin 2x = 2\sin x\cos x$$
$$\cos 2x = \cos^2 x - \sin^2 x$$
$$= 2\cos^2 x - 1$$
$$= 1 - 2\sin^2 x$$
$$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$$
$$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$$
$$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$$
$$\tan(x \pm y) = \frac{\tan x \pm \tan y}{1 \mp \tan x\tan y}$$
$$2\sin x\cos y = \sin(x+y) + \sin(x-y)$$
$$2\sin x\sin y = -\cos(x+y) + \cos(x-y)$$
$$2\cos x\cos y = \cos(x+y) + \cos(x-y)$$

Logarithm

$$a^x = e^{x\ln a}$$
$$\log_a x = \frac{\log_b x}{\log_b a}$$

Hyperbolic

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$
$$\cosh^2 x - \sinh^2 x = 1$$
$$1 - \tanh^2 x = \operatorname{sech}^2 x$$
$$\coth^2 x - 1 = \operatorname{cosech}^2 x$$
$$\sinh 2x = 2\sinh x\cosh x$$
$$\cosh 2x = \cosh^2 x + \sinh^2 x$$
$$= 2\cosh^2 x - 1$$
$$= 1 + 2\sinh^2 x$$
$$\tanh 2x = \frac{2\tanh x}{1 + \tanh^2 x}$$
$$\sinh(x \pm y) = \sinh x\cosh y \pm \cosh x\sinh y$$
$$\cosh(x \pm y) = \cosh x\cosh y \pm \sinh x\sinh y$$
$$\tanh(x \pm y) = \frac{\tanh x \pm \tanh y}{1 \pm \tanh x\tanh y}$$

Inverse Hyperbolic

$$\sinh^{-1} x = \ln(x + \sqrt{x^2+1}),\ \text{any } x$$
$$\cosh^{-1} x = \ln(x + \sqrt{x^2-1}),\ x \geq 1$$
$$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right),\ -1 < x < 1$$