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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL
COURSE CODE	:	BDU 21403
PROGRAMME CODE	:	BDC / BDM
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **FOUR (4)** QUESTIONS **ONLY.** 2. THIS FINAL EXAMINATION CONDUCTED VIA **OPEN BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

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Q1 (a) Consider a roll autopilot of a general aviation aircraft shown in **Figure Q1(a)**. Determine the value of controller gain, *K*, and parameter, p, if the step response of the aircraft is required to have an overshoot less than 5% and a settling time equal to 5 seconds. Assume $H(s) = 2$.

(10 marks)

(b) An open-loop transfer function of a unity feedback control system is given as:

$$KG(s) = \frac{K}{s(s+3)}$$

The controller gain, *K*, is to be designed such that the first peak occurs at time, $T_p = 1$ s and the percentage overshoot is 10% when excited with a step input.

(i) Can both specifications be fulfilled simultaneously? Explain your answer either with a simple mathematical proof or with an appropriate pole-zero diagram.

(5 marks)

(ii) Determine the value *K* if percentage overshoot specification can be compromised instead of peak time, T_p specification. What is the value of the maximum overshoot?

(10 marks)

Q2 (a) An aircraft's short-period response characteristics are especially important in terms of flying and handling quality. The reduced-order state-space model corresponding to the short-period mode approximation for a fixed-wing unmanned aerial vehicle (UAV) aircraft is given as follows:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & Z_q \\ M_w & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_e$$

with the following stability derivatives:

$Z_w = -7.6$ $\qquad$ $M_q = -11.5$

$Z_q = 15.5$ $\qquad$ $Z_{\delta_e} = -5.0$

$M_w = -6.1$ $\qquad$ $M_{\delta_e} = -120$

Find the solution to the state-space model using Paynter's numerical method. Use time interval, $\Delta t = 0.01$ to solve the numerical problem.

(6 marks)

(b) If the input of the system, u_1 is applied with an 1° elevator step input with an output equation and initial condition given as follows:

$$q_k = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} w_k \\ q_k \end{bmatrix}$$

$$\begin{bmatrix} w_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.15 \end{bmatrix}$$

Determine the output response, q_k of the state equation for three (3) iterations.

(6 marks)

(c) Examine the stability derivatives' influence on the damping ratio and natural frequency of the short-period motion.

(5 marks)

(d) Comment on the time response characteristics of the short-period motion obtained for this aircraft. Do your findings agree with the handling quality criteria shown in **Figure Q2(d)**?

(8 marks)

Q3 (a) Describe the physical features of the Dutch Roll stability mode.

(3 marks)

(b) The Dutch Roll motion can be approximated using the following equation:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & -\left(1 - \frac{Y_r}{u_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta r}}{u_0} \\ N_{\delta r} \end{bmatrix} \delta r$$

Assume the aircraft has the following stability derivative characteristics as follows:

$Y_\beta = -6.5$ ft/s^2	$Y_r = 1.5$ ft/s
$N_\beta = 4.0$ s^{-2}	$N_r = -0.5$ s^{-1}
$Y_{\delta r} = -4.5$ ft/s^2	$N_{\delta r} = -150.515$ s^{-2}
$u_0 = 100$ ft/s	

(i) Determine the characteristic equation of the Dutch Roll mode.

(5 marks)

(ii) Determine the eigenvalues of the Dutch Roll mode.

(2 marks)

(iii) Determine the damping ratio, natural frequency, period, time to half amplitude, and the number of cycles to half amplitude for the Dutch Roll mode.

(5 marks)

(c) A helicopter based unmanned aerial vehicle (UAV), as shown in **Figure Q3(c)** is an unmanned aircraft that uses its tail rotor to change its heading direction. The yaw angle to the rudder input transfer function of the helicopter UAV can be modeled according to:

$$\frac{\psi(s)}{\delta_{rud}(s)} = \frac{2}{s^2 + 0.1s + 5}$$

Design a heading control system so that the UAV can exhibit yaw angle tracking performance with a desired damping ratio of $\xi = 0.7$, setting time, $t_s \leq 3$ s and no steady state error. Consider the sensor used in the control system design to be a perfect device.

(10 marks)

Q4 An Ultrastick 25e UAV platform with a 1.27 m wingspan and a gross weight of 1.9 kg has the following transfer function for pitch angle to the elevator input:

$$G(s) = \frac{s^2 + 13s + 20}{s^4 + 12.2s^3 + 35.41s^2 + 18.52s + 32.3}$$

The pitch control system that will be designed for the UAV platform is shown in **Figure Q4** with transfer functions for controller gain given as:

$$K(s) = K_P + \frac{K_I}{s} + K_D s$$

(a) Consider the controller's transfer function is set with proportional gain only, examine the closed-loop pole movement of the pitch control system if the controller gain varies from 0 to ∞. Discuss whether the Ziegler and Nichols tuning method is a suitable tuning method to be used with the pitch control system. Determine the damped frequency, ω_d and gain, K, values at the imaginary axis crossing if such a situation exists.

(10 marks)

(b) Develop the automatic controllers (i.e., P, PD, and PID control) for the dynamic system under consideration using the Ziegler and Nichols tuning method. Compare the steady-state error performances of the compensated systems (i.e., P, PD, and PID control). Describe any problems with your design.

(15 marks)

Q5 (a) A high-performance aircraft shown in **Figure Q5(a)** uses the ailerons, elevator, rudder, lift fan, and thrust vector nozzle to maneuver through a three-dimensional flight path. The pitch angle control system for the high-performance aircraft at 15,000 m and Mach 0.9 can be represented by the block diagram in **Figure Q5(b)**. The pitch angle to elevator transfer function for the aircraft is given as follows:

$$G(s) = \frac{(s + 6)}{s(s + 4)(s^2 + 4s + 8)}$$

Suggest a value of gain, *K*, that results in a system with a damping ratio for the complex roots equal to 0.5. Provide a detailed root locus plot for the closed-loop system as *K* varies from 0 to ∞ with the necessary calculation to support your answer. Determine the settling time and overshoot for the system when the damping ratio of the complex roots is equal to 0.5.

(20 marks)

(b) Comment on whether the current performance can be improved further. Suggest the value of controller gain, *K*, with the maximum achievable value of the damping ratio. Compare the settling time and overshoot obtained with the time response performance in **Q5(a)**.

(5 marks)

-END OF QUESTION-

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FINAL EXAMINATION

SEMESTER/SESSION	:	SEM I / 2022/2023	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

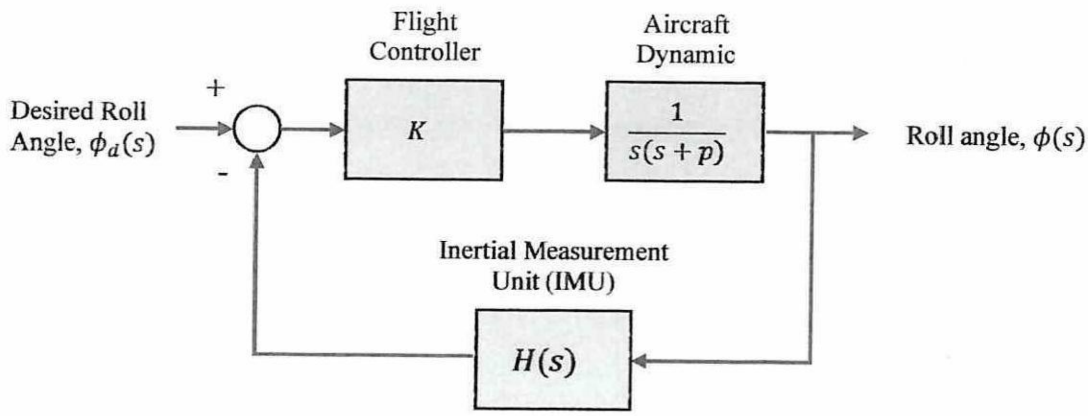

Figure Q1(a) Roll angle control system.

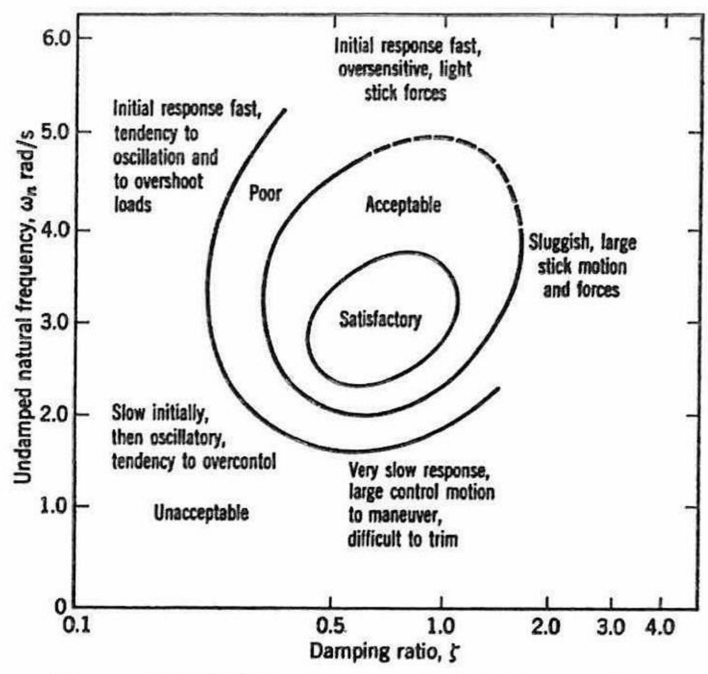

Figure Q2(d) The short period flying quality.

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM I / 2022/2023	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

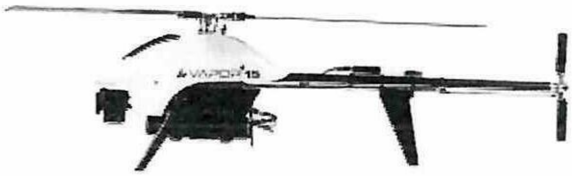

Figure Q3(c) Helicopter based unmanned aerial vehicle (UAV).

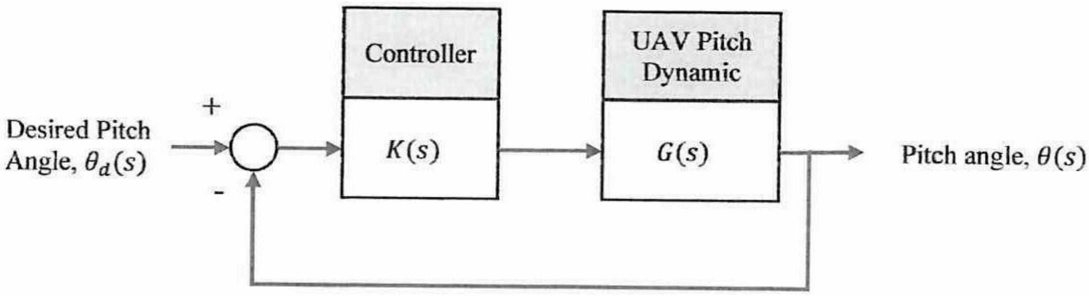

Figure Q4 Simplified block diagram for pitch angle control system.

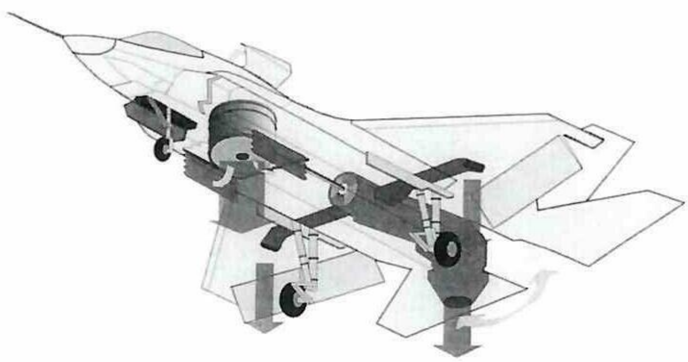

Figure Q5(a) The high-performance aircraft.

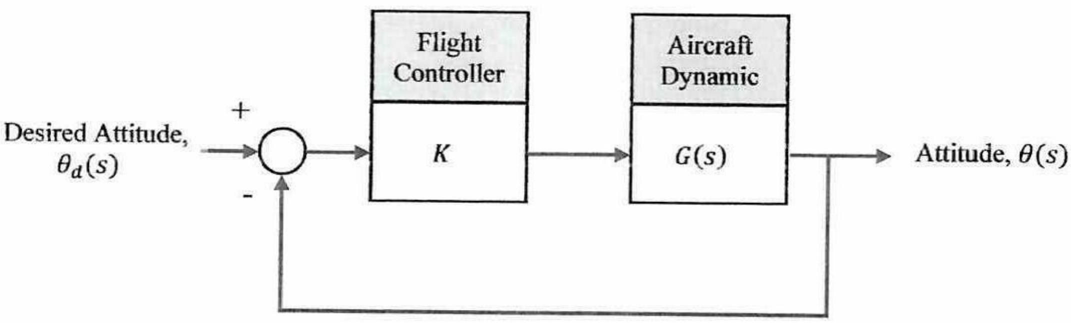

Figure Q5(b) The block diagram for the pitch control system.

FINAL EXAMINATION

SEMESTER/SESSION	: SEM I / 2022/2023	PROGRAMME CODE	: BDC/BDM
COURSE NAME	: AIRCRAFT STABILITY AND CONTROL	COURSE CODE	: BDU 21403

Key Equations

The relevant equations used in this examination are given as follows:

1. The determinant of a 3×3 matrix:
$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad (1)$$
2. Partial fraction for $F(s)$ with real and distinct roots in the denominator:
$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2}{(s+p_2)} + \cdots + \frac{K_m}{(s+p_m)} \quad (2)$$
3. Partial fraction for $F(s)$ with complex or imaginary roots in the denominator:
$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2+as+b)} + \cdots \quad (3)$$
4. General first-order transfer function:
$$G(s) = \frac{K}{s+a} \quad (4)$$
5. General second-order transfer function:
$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n^2} \quad (5)$$
6. The closed-loop transfer function:
$$T(s) = \frac{G(s)}{1 + G(s)H(s)} \quad (6)$$
where $G(s)$ is the transfer function of the open-loop system, and $H(s)$ is the transfer function in the feedback loop.
7. The final value theorem:
$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (7)$$
8. Time response:
$$T_r = \frac{2.2}{a} \quad (8)$$
$$T_s = \frac{4}{a} \quad (9)$$
$$\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\% \quad (10)$$
$$\xi = \frac{-\ln\left(\%\frac{OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\%\frac{OS}{100}\right)\right)^2}} \quad (11)$$

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM I / 2022/2023	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega_d} \quad (12)$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\sigma} \quad (13)$$

$$P = \frac{2\pi}{\omega_d} \quad (14)$$

$$t_{1/2} = \frac{0.693}{|\sigma|} \quad (15)$$

$$N_{1/2} = 0.110\frac{|\omega_d|}{|\sigma|} \quad (16)$$

9. Estimation of parameter q (Paynter Numerical Method)

$$q = \max\left|A_{ij}\Delta t\right| \quad (17)$$

10. Estimation of the integer value of p (Paynter Numerical Method)

$$\frac{1}{p!}(nq)^p e^{nq} \leq 0.001 \quad (18)$$

11. Numerical solution of state equation:

$$\mathbf{x}_{k+1} = \boldsymbol{M}\mathbf{x}_k + \boldsymbol{N}\boldsymbol{u}_k$$

with matrix $\boldsymbol{M}$ and $\boldsymbol{N}$ are given by the following matrix expansion:

$$\boldsymbol{M} = e^{A\Delta t} = \boldsymbol{I} + \boldsymbol{A}\Delta t + \frac{1}{2!}\boldsymbol{A}^2\Delta t^2 \ldots \quad (19)$$

$$\boldsymbol{N} = \Delta t\left(\boldsymbol{I} + \frac{1}{2!}\boldsymbol{A}\Delta t + \frac{1}{3!}\boldsymbol{A}^2\Delta t^2 + \cdots\right)\boldsymbol{B}$$

12. Characteristic equation of the closed loop system:

$$1 + KG(s)H(s) = 0 \quad (20)$$

13. Asymptotes: angle and real-axis intercept:

$$\sigma = \frac{[\sum Real\ parts\ of\ the\ poles - \sum Real\ parts\ of\ the\ zeros]}{n-m} \quad (21)$$

$$\phi_a = \frac{180°[2q+1]}{n-m} \quad (22)$$

14. The solution to determine real axis break-in and breakaway points:

$$\frac{dK(s)}{ds} = 0 \quad (23)$$

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM I / 2022/2023	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU21403

15. An alternative solution to find real axis break-in and breakaway points:

$$\sum \frac{1}{s+z_i} = \sum \frac{1}{s+p_i} \tag{24}$$

16. The angle of departure of the root locus from a pole of $G(s)H(s)$:

$$\theta = 180^\circ + \sum (angles\ to\ zeros) - \sum (angles\ to\ poles) \tag{25}$$

17. The angle of arrival at a zero:

$$\theta = 180^\circ - \sum (angles\ to\ zeros) + \sum (angles\ to\ poles) \tag{26}$$

18. The steady-state error:

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s) \tag{27}$$

where the error signal is given as:

$$E(s) = \frac{1}{1+K(s)G(s)H(s)} \times U(s) \tag{28}$$

19. The characteristic equation for the standard form of the second-order differential equation:

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0$$

The roots of the characteristic equation are: (29)

$$s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1-\xi^2}\cdot i$$

$$s_{1,2} = \sigma \pm \omega_d$$

20. The calculation of controller gains using the Ziegler-Nichols method:

Table 1 The Ziegler-Nichols tuning method. (30)

Control Type	K_p	K_I	K_D
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2\,K_p/T_u$	-
PD	$0.8K_u$	-	$K_P T_u/8$
Classic PID	$0.6K_u$	$2\,K_p/T_u$	$K_P T_u/8$
Pessen Integral Rule	$0.7K_u$	$2.5\,K_p/T_u$	$3K_P T_u/20$
Some Overshoot	$0.33K_u$	$2\,K_p/T_u$	$K_P T_u/3$
No Overshoot	$0.2K_u$	$2\,K_p/T_u$	$K_P T_u/3$

21. Conversion from the state-space model to transfer function model: (31)

$$G(s) = \boldsymbol{C}\frac{adj(\boldsymbol{sI}-\boldsymbol{A})}{\det(\boldsymbol{sI}-\boldsymbol{A})}\boldsymbol{B}$$