Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	CALCULUS FOR ENGINEER
COURSE CODE	:	BDA 14403
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. PART A: ANSWER **ALL** QUESTIONS. PART B: ANSWER **TWO (2)** FROM **THREE (3)** QUESTIONS ONLY. 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK.** 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

PART A:

Q1 (a) Find the derivative of the given function by using the appropriate formula

(i) $f(x) = (x^{100} + 2x^{50} - 3)(7x^8 + 20x + 5)$.

(2 marks)

(ii) $g(x) = \frac{x^5 - x + 2}{x^3 + 7}$.

(4 marks)

(b) Oil from an uncapped well is radiating outward in the form of a circular film on the surface of the water. If the radius of the circle is increasing at the rate of 1.5 meters per minute, how fast is the area of the oil film growing at the instant when the radius is 150 meter?

(4 marks)

(c) Given that $f(x, y, z) = \cos(x^3y + z^2)$, where $x = sr^2,\ y = r^3 + 5s,\ Z = r^5$. Find $\frac{\delta f}{\delta s}$.

(5 marks)

(d) Find the slope of the surface given by $f(x, y) = \frac{-x^5}{5} - 3xy^2 + \frac{3}{8}$ at the point (1, 2, 3) in the x – direction and y – direction.

(5 marks)

Q2 (a) Solve the following by using polar coordinates

$$\int_0^3 \int_0^{\sqrt{9-x^2}} \frac{r\cos\theta}{\sqrt{(r\cos\theta)^2 + (r\sin\theta)^2}}\, dy\, dx.$$

(8 marks)

(b) Find the volume of the solid tetrahedron enclosed by $2x + y + z = 4$ and the coordinate planes by using triple integral.

(12 marks)

Q3 (a) Find the directional derivatives for $f(x, y) = xy^2 + 3ye^{2x}$ at (0,1) in the direction of vector $\mathbf{a} = 4i + 3j$.

(6 marks)

(b) Given the vector field of $\mathbf{F}(x, y, z) = e^{-xy}i + e^{xz}j + e^{yz}k$. Find the following;

(i) The divergence at (3, 2, 0).

(3 marks)

(ii) The curl at (3, 2, 0).

(5 marks)

(c) Solve $\int_C \frac{1}{1+x}\, ds$ where $C: x = t, y = \frac{2}{3}t^{3/2}, 3 \le t \le 8$.

(6 marks)

PART B:

Q4 (a) Solve the integral $\int_C (x^2 - y^2)dx - 2xy\,dy$ along the parabola $y = 2x^2$ from $(0,0)$ to $(1,2)$.

(8 marks)

(b) Use Green's Theorem to evaluate $\int_C (2xy - y^2)dx + (x^2 - y^2)dy$, where C is the boundary of the region enclosed by $y = x$ and $y = x^2$. Assume that the curve c is traversed in a counter clockwise manner.

(8 marks)

(c) Given $\mathbf{F}(x, y, z) = x^4 y^2\,\mathbf{i} + y^2 z\,\mathbf{j} + xz^3\mathbf{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$. Find the integrand that derived when using Gauss's Theorem to evaluate $\iint_S \mathbf{F}\cdot\mathbf{n}\,dS$. **(Do not solve the integration)**

(4 marks)

Q5 (a) Find the domain and the range for function $f(x, y) = \sqrt{9 - 3x^2 - 4y^2}$. Then sketch the graph.

(8 marks)

(b) Given $f(x, y) = \sqrt{4 - x^2 - y^2}$. Show that $f_{xy} = f_{yx}$.

(9 marks)

(c) Find the limit of the function $f(x, y) = \frac{yx^4 - y^5}{x^2 + y^2}$ when $(x, y) \to (0,0)$.

(3 marks)

Q6 (a) By using double integral, find the volume of solid enclosed by cylinder $y^2 + x^2 = 1$, plane $z = y$ and *yz*-plane in the first octant.

(6 marks)

(b) By using spherical coordinate, find the volume of the solid bounded side by sphere $x^2 + y^2 + z^2 = 16$ and above by cone $z = \sqrt{x^2 + y^2}$ below plane $z = 0$.

(8 marks)

(c) Solve $\int_c xydx + (x^2 + y^2)dy$, where C is a straight line segment from (0,0) to (2,4).

(6 marks)

-END OF QUESTIONS –

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FORMULA

Total Differential

For function $z = f(x,y)$, the total differential of z, dz is given by:

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Relative Change

For function $z = f(x,y)$, the relative change in z is given by:

$$\frac{dz}{z} = \frac{\partial z}{\partial x}\frac{dx}{z} + \frac{\partial z}{\partial y}\frac{dy}{z}$$

Rate of Change

For function with single variable, $y(x)$, the rate of change is given by

$$\frac{dy}{dx} = \frac{dy}{dx}\frac{dx}{dt}$$

For function with two variables, $z = f(x,y)$, the rate of change is given by

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

Increment of Approximation

For a function of two variables, , $z = f(x,y)$, when , (x,y) moves from the initial point , (x_0, y_0) to (x_1, y_1), the following can be calculated by using total differential and partial derivative;

Approximate change in z;

$$dz = \frac{\partial z}{\partial x}dx + \frac{\partial z}{\partial y}dy$$

Exact change in z;

$$\Delta z = f(x_1, y_1) - f(x_0, y_0)$$

Approximate value of z;

$$z = f(x_0, y_0) + dz$$

Exact value of z;

$$z = f(x_1, y_1)$$

Implicit Differentiation

Suppose that z is given implicitly as a function $z = f(x,y)$ by an equation of the form $F(x,y,z) = 0$, where $F(x,y,f(x,y)) = 0$ for all (x,y) in the domain of *f*, hence,

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} \text{ and } \frac{\partial z}{\partial y} = -\frac{F_y}{F_z}$$

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Extreme of Function with Two Variables

$D = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$

a. If $D > 0$ and $f_{xx}(a,b) < 0$ (or $f_{yy}(a,b) < 0$)
   $f(x,y)$ has a local maximum value at (a,b)

b. If $D > 0$ and $f_{xx}(a,b) > 0$ (or $f_{yy}(a,b) > 0$)
   $f(x,y)$ has a local minimum value at (a,b)

c. If $D < 0$
   $f(x,y)$ has a saddle point at (a,b)

d. If $D = 0$
   The test is inconclusive

Surface Area

$$\text{Surface Area} = \iint_R dS = \iint_R \sqrt{(f_x)^2 + (f_y)^2 + 1}\, dA$$

Polar Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

where $0 \le \theta \le 2\pi$

$$\iint_R f(x,y)\,dA = \iint_R f(r,\theta)\,r\,dr\,d\theta$$

Cylindrical Coordinates:

$x = r\cos\theta$

$y = r\sin\theta$

$z = z$

where $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(r,\theta,z)\,r\,dz\,dr\,d\theta$$

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Spherical Coordinates:

$x = \rho \sin\phi \cos\theta$

$y = \rho \sin\phi \sin\theta$

$z = \rho \cos\phi$

$\rho^2 = x^2 + y^2 + z^2$

where $0 \le \phi \le \pi$ and $0 \le \theta \le 2\pi$

$$\iiint_G f(x,y,z)\,dV = \iiint_G f(\rho,\phi,\theta)\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

In 2-D: Lamina

Given that $\delta(x,y)$ is a density of lamina

Mass, $m = \iint_R \delta(x,y)\,dA$, where

Moment of Mass

a. About x-axis, $M_x = \iint_R y\delta(x,y)\,dA$

b. About y-axis, $M_y = \iint_R x\delta(x,y)\,dA$

Centre of Mass

Non-Homogeneous Lamina:

$$(\bar{x},\bar{y}) = \left(\frac{M_y}{m}, \frac{M_x}{m}\right)$$

Centroid

Homogeneous Lamina:

$$\bar{x} = \frac{1}{Area\ of\ R}\iint_R x\,dA \text{ and } \bar{y} = \frac{1}{Area\ of\ R}\iint_R y\,dA$$

Moment Inertia:

a. $I_y = \iint_R x^2\delta(x,y)\,dA$

b. $I_x = \iint_R y^2\delta(x,y)\,dA$

c. $I_o = \iint_R (x^2+y^2)\delta(x,y)\,dA$

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In 3-D: Solid

Given that $\delta(x, y, z)$ is a density of solid

Mass, $m = \iiint_G \delta(x, y, z) dV$

If $\delta(x, y, z) = c$, where c is a constant, $m = \iiint_G dA$ is volume.

Moment of Mass

a. About yz-plane, $M_{yz} = \iiint_G x\delta(x, y, z) dV$

b. About xz-plane, $M_{xz} = \iiint_G y\delta(x, y, z) dV$

c. About xy-plane, $M_{xy} = \iiint_G z\delta(x, y, z) dV$

Centre of Gravity

$(\bar{x}, \bar{y}, \bar{z}) = \left(\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m} \right)$

Moment Inertia

a. About x-axis, $I_x = \iiint_G (y^2 + z^2)\delta(x, y, z) dV$

b. About y-axis, $I_y = \iiint_G (x^2 + z^2)\delta(x, y, z) dV$

c. About z-axis, $I_z = \iiint_G (x^2 + y^2)\delta(x, y, z) dV$

Directional Derivative

$D_u f(x, y) = (f_x \mathbf{i} + f_y \mathbf{j}) \cdot \mathbf{u}$

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Del Operator

$$\nabla = \frac{\partial}{\partial x}\mathbf{i} + \frac{\partial}{\partial y}\mathbf{j} + \frac{\partial}{\partial z}\mathbf{k}$$

Gradient of $\phi = \nabla \phi$

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$, hence,

The **Divergence** of $\mathbf{F} = \nabla . \mathbf{F} = \dfrac{\partial M}{\partial x} + \dfrac{\partial N}{\partial y} + \dfrac{\partial P}{\partial z}$

The **Curl** of $\mathbf{F} = \nabla \times \mathbf{F}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \dfrac{\partial}{\partial x} & \dfrac{\partial}{\partial y} & \dfrac{\partial}{\partial z} \\ M & N & P \end{vmatrix}$$

$$= \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let C is smooth curve defined by $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, hence,

The **Unit Tangent Vector,** $\mathbf{T}(t) = \dfrac{\mathbf{r}'(t)}{\| \mathbf{r}'(t) \|}$

The **Principal Unit Normal Vector,** $\mathbf{N}(t) = \dfrac{\mathbf{T}'(t)}{\| \mathbf{T}'(t) \|}$

The **Binormal Vector**, $\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$

Curvature

$$\kappa = \frac{\| \mathbf{T}'(t) \|}{\| \mathbf{r}'(t) \|}$$

Radius of Curvature

$$\rho = \frac{1}{\kappa}$$

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Green's Theorem

$$\oint_C M\,dx + N\,dy = \iint_R \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} \right) dA$$

Gauss's Theorem

$$\iint_S \mathbf{F} \cdot \mathbf{n}\,dS = \iiint_G \nabla \cdot \mathbf{F}\,dV$$

Stoke's Theorem

$$\oint_C \mathbf{F} \cdot dr = \iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\,dS$$

Arc Length

If $\mathbf{r}(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}, t \in [a,b]$, hence, the **arc length**,

$$s = \int_a^b \| \mathbf{r}'(t) \| \,dt = \int_a^b \sqrt{[x'(t)]^2 + [y'(t)]^2 + [z'(t)]^2}\,dt$$