Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	ENGINEERING TECHNOLOGY MATHEMATICS II
COURSE CODE	:	BDU 11003
PROGRAMME CODE	:	BDM
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN **PART A** AND **THREE (3)** QUESTIONS IN **PART B**. 2. THIS FINAL EXAMINATION IS A CONDUCTED **CLOSED BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

PART A

Q1 A periodic function $f(x)$ is defined by

$$f(x) = \begin{cases} 1, & -1 < x < 0 \\ 3, & 0 < x < 1 \end{cases}$$

and $f(x) = f(x+2)$.

(a) Sketch the graph of $f(x)$ over $-3 < x < 3$.

(2 marks)

(b) Find the Fourier coefficients corresponding to $f(x)$.

(13 marks)

(c) From (b), prove that the Fourier series for $f(x)$

$$f(x) = 2 + \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\sin(2n-1)\pi x}{2n-1}.$$

(5 marks)

Q2 (a) Solve the system of linear equations below by Gauss elimination method.

$$\begin{aligned} 2x_1 + 3x_2 + 10x_3 &= 9 \\ 5x_1 - x_2 + 3x_3 &= 6 \\ 4x_1 + 7x_2 + x_3 &= 2. \end{aligned}$$

(10 marks)

(b) Solve the system of linear equations by using Thomas algorithm method.

$$\begin{pmatrix} 1 & 2 & 0 & 0 \\ 3 & 4 & 5 & 0 \\ 0 & 6 & 7 & 8 \\ 0 & 0 & 10 & 11 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 3 \end{pmatrix}.$$

(10 marks)

PART B

Q3 (a) By using the method of undetermined coefficients, solve

$$y'' - 2y' + y = x^2 - 3x + \sin 2x$$

with $y(0) = 0$ and $y'(0) = 1$.

(13 marks)

(b) Find the general solution of the equation

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = \frac{e^x}{x}$$

using the method of variation of parameters.

(7 marks)

Q4 (a) Determine the Laplace transform for each of the following function:

(i) $f(t) = t^4 + 2t^3 - 4t^2 + t + 1$.
(ii) $f(t) = (2t - e^{2t})^2$.
(iii) $f(t) = 3e^{-2t} \sin 5t$.

(10 marks)

(b) Consider the step function

$$f(t) = \begin{cases} 2(1-t), & 0 \le t < 1 \\ 2(t-1), & 1 \le t < 2 \\ 2, & t \ge 2. \end{cases}$$

Sketch the graph of $f(t)$, express $f(t)$ in terms of unit step functions and find its Laplace transform.

(10 marks)

Q5 (a) Solve the initial value problem by using the inverse Laplace transform

$$y'' + 4y = e^{-t}$$

with initial value $y(0) = 2$ and $y'(0) = 1$.

(8 marks)

(b) (i) Express

$$\frac{1}{(s-1)(s-2)^2}$$

in partial fractions and show that

$$\mathcal{L}^{-1}\left\{\frac{1}{(s-1)(s-2)^2}\right\} = e^t - e^{2t} + te^{2t}.$$

(ii) Use the result in **Q5(b)(i)** to solve the differential equation

$$y' - y = te^{2t}$$

which satisfies the initial condition of $y(0) = 1$.

(12 marks)

Q6 (a) (i) Show that

$$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$$

is a homogeneous differential equation.

(ii) Hence, solve the differential equation in part **Q6(a)(i)**.

(12 marks)

(b) Choose an appropriate method to solve the following problem and obtain its particular solution

$$\frac{dy}{dx} + y\tan x = \cos^2 x, y(0) = 1.$$

(8 marks)

-END OF QUESTIONS-

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Formula's

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W}dx, \quad u_2 = \int \frac{y_1 f(x)}{W}dx$	$y_p = u_1 y_1 + u_2 y_2$

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Laplace Transforms

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
$t^n,\ n=1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$f(t)H(t-a)$	$e^{-as}F(s+a)$
$\sin at$	$\frac{a}{s^2+a^2}$	$\delta(t-a)$	e^{-as}
$\cos at$	$\frac{s}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$y(t)$	$Y(s)$
$e^{at}f(t)$	$F(s-a)$	$\dot{y}(t)$	$sY(s)-y(0)$
$t^n f(t),\ n=1, 2, 3, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$

Fourier Series

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi x}{L} + b_n \sin\frac{n\pi x}{L}\right]$$

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

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Thomas Algorithm

i	1	2	…	n
d_i				
e_i				
c_i				
b_i				
$\alpha_1 = d_1$ $\alpha_i = d_i - c_i\beta_{i-1}$				
$\beta_i = \dfrac{e_i}{\alpha_i}$				
$y_1 = \dfrac{b_1}{\alpha_1}$ $y_i = \dfrac{b_i - c_i y_{i-1}}{\alpha_i}$				
$x_n = y_n$ $x_i = y_i - \beta_i x_{i+1}$				