UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	NUMERICAL METHOD
COURSE CODE	:	BDA 34103
PROGRAMME CODE	:	BDD
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS IN **PART A**. 2. ANSWER **ONE (1)** QUESTION ONLY IN **PART B**. 3. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 4. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

PART A: ANSWER ALL QUESTIONS

Q1 The problem of transient radial heat flow in a circular rod in nondimensional form is described by:

$$\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}=\frac{\partial T}{\partial t}$$

where:

r = radius of the circular rod,
t = time, and
T= temperature.

Boundary conditions : $T(1,t)=1$ and $T(0,t)=1$
Initial conditions : $T(r,0)=1$ and $0\leq r\leq 1$

Implicit Crank-Nicolson method is decided to approximate the temperature distribution along the radial direction of the rod. The value of Δr and Δt is set to 0.25 and 0.01, respectively. The implicit Crank-Nicolson equation for the problem can be written as:

$$(0.02r+0.0025)\,T_{(r+1,t+1)}+(-0.29r)\,T_{(r,t+1)}+(0.02r-0.0025)\,T_{(r-1,t+1)}=$$
$$-(0.02r+0.0025)\,T_{(r+1,t)}+(-0.21r)\,T_{(r,t)}-(0.02r-0.0025)\,T_{(r-1,t)}$$

(a) Draw the computational molecule for this problem.

(4 marks)

(b) Draw the finite difference grid to predict the temperature of all points up to t = 0.02. Set values of Δr and Δt equal to 0.25 and 0.01, respectively.

(6 marks)

(c) Write and transform the finite difference equations for t = 0.01 to a system of linear equations.

(9 marks)

(d) Transform your answer in Q1(c) into matrix form.

(1 marks)

Q2 (a) **Figure Q2(a)** illustrates the difference between Initial Value Problem (IVP) and Boundary Value Problem (BVP). With short explanation, discuss the difference between Initial Value Problem (IVP) and Boundary Value Problem (BVP).

(4 marks)

(b) A ball at 1200K is allowed to cool down in air at an ambient temperature of 300K. Assuming heat is lost only due to radiation, the differential equation for the temperature of the ball is given by

$$\frac{d\theta}{dt}=-2.2067\times 10^{-12}(\theta^4-81\times 10^8),\ \theta(0)=1200\text{K}$$

where θ is in K and t in seconds.

i) Find the temperature at $0 < t < 1200\ seconds$ using Euler's method. Assume a step size of $h = 120\ seconds$.

(10 marks)

ii) **Figure Q2(b)** shows the comparison between estimated solution (with an increase step size, h) towards the exact solution. If the step size is increased to $h = 240\ seconds$, re-determine the temperature at $0 < t < 1200\ seconds$ using Euler's method. Highlight which step size ($h = 120\ seconds$ or $h = 240\ seconds$) that will result in better solution (in terms of accuracy)? Use **Figure Q2(b)** to support your answer.

(6 marks)

Q3 The characteristic equation of a 3 degree of freedom spring mass system as shown in **Figure Q3** can be further developed as a set of simultaneous equations:

$$5V_1 - 3V_2 = 0$$
$$-3V_1 + 4V_2 - V_3 = 0$$
$$-V_2 + 6V_3 = 0$$

(a) Write the simultaneous equation above in a complete matrix form of $[A][V] = 0$.

(2 marks)

(b) Determine the largest eigenvalue and its corresponding eigenvector using Power Method. Use the initial eigenvector $[V] = (1 \quad 2 \quad 1)^T$ and stop the iteration when $(|\lambda_{i+1} - \lambda_i| < 0.3)$. Perform your calculation in 4 decimal points.

(8 marks)

(c) Determine the smallest eigenvalue and its corresponding eigenvector using Inverse Power Method. Use the initial eigenvector $[V] = (1 \quad 2 \quad 1)^T$ and stop the iteration when $(|\lambda_{i+1} - \lambda_i| < 0.001)$. Perform your calculation in 4 decimal points.

(10 marks)

Q4 (a) Zaafran is an engineer in Ferrari team. He develops a car simulation on the track which can be expressed in a polynomial function as:

$$v(t) = 60t^3 + 98t^2 + 54t$$

where velocity, v is measured in m/s and time, t in seconds. If the car starts moving from starting point (0 seconds) to the end point (9 seconds), calculate the distance travelled by the car within 9 seconds from the starting point (consider $h = 1$), by using:

(i) Simpson's 3/8 rule.

(4 marks)

(ii) 3 Point Gauss quadrature.

(6 marks)

(b) A new bullet test for bullet proof jacket has detected a motion equation as follows:

$$s(t) = (10 + 12t)e^{-0.3t}$$

By using $h = 0.01$, determine

(i) The velocity of the object at 25 second by using 3-Point Central Difference.
(4 marks)

(ii) The acceleration of the object at 15 second by using 3-Point Central Difference.
(6 marks)

PART B: ANSWER ONE (1) QUESTION ONLY

Q5 The stress concentration factor, K for a flat bar with a centric hole under axial loading is:

$$K = 3.00 - 3.13\left(\frac{2r}{D}\right) + 3.66\left(\frac{2r}{D}\right)^2 - 1.53\left(\frac{2r}{D}\right)^3$$

where:

r = radius of the hole, and
D = the width of the bar,

If D = 75 mm, approximate the value of r to obtain K = 3.5.

(a) Determine your result using Bisection and Secant method. All data /values shall be in 4-decimal place. Do iteration up to five (5) iteration or stop if two consecutive r values are less than 0.001. (Hint: r is somewhere between 4 and 6 mm. Use this value as initial guess)

(17 marks)

(b) Discuss the differences of the answers from **Q5(a)**.

(3 marks)

Q6 (a) Two cars are on the road. The position of the first car at any instant in time is given by:

$$s_1(t) = t^2 - 1$$

and the position of the second car at any instant of time is given by:

$$s_2(t) = \cos(t)$$

Estimate the time when both cars arrive at the same position using Secant method. Start with the interval [0.5,1] and iterate until $|t_{i+1} - t_i| \leq 0.05$.

(10 marks)

(b) The differential equation for steady state condition of heat conduction through a wall with considering internal heat generation is given by:

$$\frac{d^2T}{dx^2} + \frac{G}{k} = 0$$

where T = temperature (°C), x = position (cm), G = internal heat source (W/cm^3), and k = thermal conductivity (W/cm/°C). An experimental work was done and the temperatures of the wall at specific positions were measured and tabulated in **Table Q6(a)**.

Table Q6(a): Wall temperature at different positions.

x (cm)	-3.00	-2.25	-1.50	-0.75	0.75	1.50	2.25	3.00
T (°C)	40.00	42.81	44.82	46.03	46.03	44.82	42.81	40.00

The Newton's divided difference interpolation polynomial is given by:

$$T(x) = -0.7170x^2 + 46.4334$$

By considering the data from $x = -1.5$ to $x = 1.5$, interpolate the temperature at $x = 0$ by using Newton's divided difference.

(10 marks)

-END OF QUESTION-

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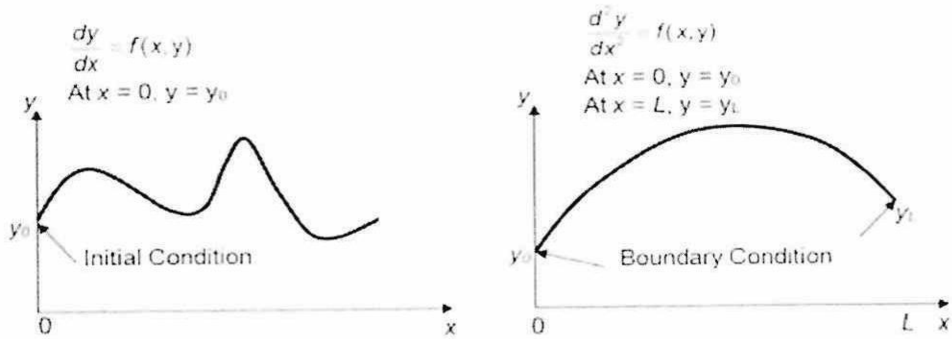

Figure Q2(a)

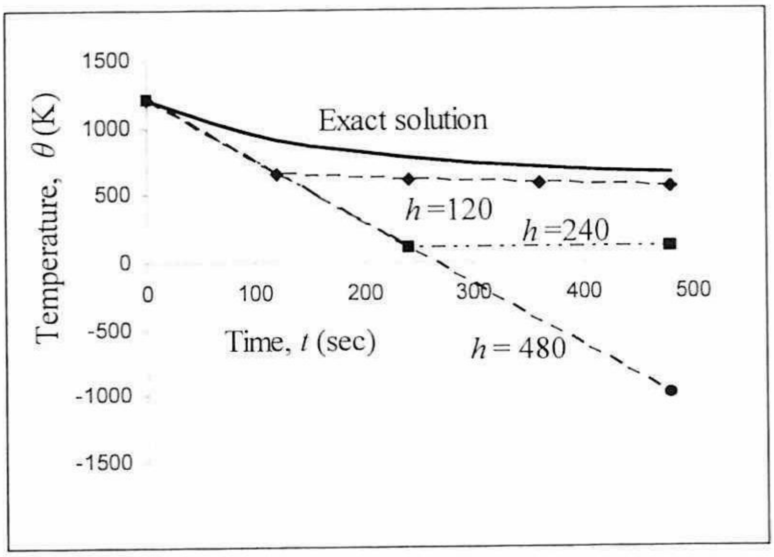

Figure Q2(b)

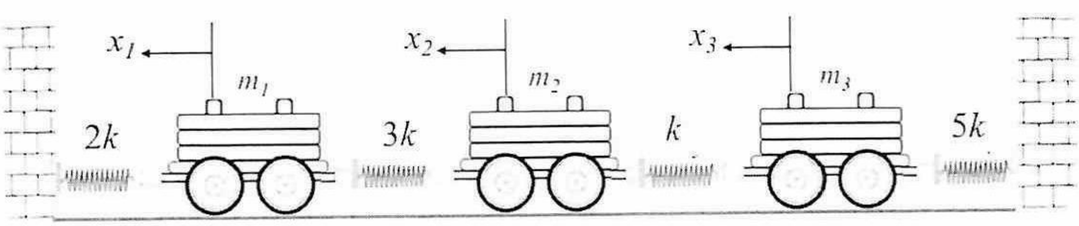

Figure Q3

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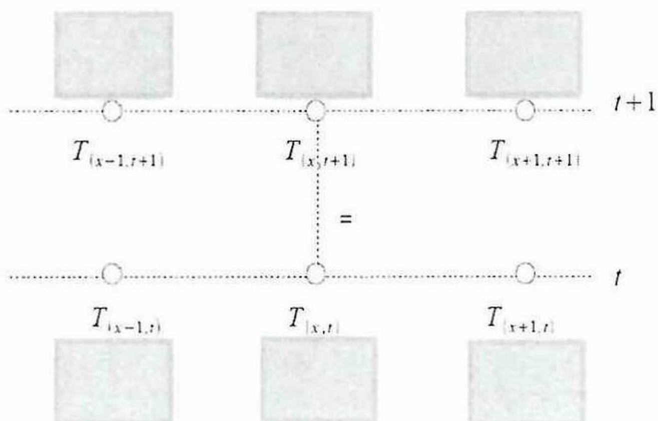
FORMULA

Implicit Crank Nicolson Method:



Euler 's Method:

$$y(x_{i+1}) = y(x_i) + y'(x_i)\,h$$

Power Method:

$$\{V\}^{k+1} = \frac{[A]\{V\}^{k}}{\lambda_{k+1}}$$

Inverse Power Method:

$$\{V\}^{k+1} = \frac{[A]^{-1}\{V\}^{k}}{\lambda_{k+1}}$$

Simpson 3/8:

$$\int_{x_1}^{x_n} y(x)dx = \frac{3h}{8}\left(y_1 + 3\,(y_2 + y_3 + y_5 + y_6 + \cdots) + 2(y_4 + y_7 + \cdots) + y_n\right)$$

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Gauss Quadrature:

$$x_\xi = \frac{1}{2}[(1-\xi)x_0 + (1+\xi)x_n]$$

$$I = \left(\frac{x_n - x_0}{2}\right) I_\xi$$

$$I_\xi = R_1\phi(\xi_1) + R_2\phi(\xi_2) + \cdots + R_n\phi(\xi_n)$$

n	$\pm\xi_j$	R_j
1	0.0	2.0
2	0.5773502692	1.0
3	0.7745966692	0.555555556
	0.0	0.888888889

3 Point Central Difference:

$$y'(x) = \frac{y(x+h) - y(x-h)}{2h}$$

$$y''(x) = \frac{y(x+h) - 2y(x) + y(x-h)}{h^2}$$

Bisection Method:

$$c = \frac{a+b}{2}$$

Secant Method:

$$x_{i+1} = \frac{x_{i-1}y(x_i) - x_i y(x_{i-1})}{y(x_i) - y(x_{i-1})}$$

Newton Divided Difference:

$$y(x) = a_0 + a_1(x-x_0) + a_2(x-x_0)(x-x_1) + \ldots + a_n(x-x_0)(x-x_1)\ldots(x-x_{n-1})$$