Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	CALCULUS
COURSE CODE	:	DAS 20803
PROGRAMME CODE	:	DAU
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS
		2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**.
		3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) Sketch the graph of the following functions, hence determine the domain and range:

$$f(x) = \begin{cases} \dfrac{1}{(2x-1)^2} & ; x \leq 1 \\ x^2 - 2x + 3 & ; x > 1 \end{cases}.$$

(7 marks)

(b) Given the function, $f(x) = 10^{4x}$.

(i) Determine the inverse function, $f^{-1}(x)$.

(3 marks)

(ii) Hence, sketch the graph of $f(x) = 10^{4x}$ and its inverse.

(3 marks)

(c) If $f(x) = 3x + 2$, $g(x) = 5^{-x}$ and $h(x) = 1 - x^2$. Calculate:

(i) $(f \circ g)(x)$.

(2 marks)

(ii) $(f \circ g \circ h)(4)$.

(5 marks)

Q2 (a) Evaluate the following limit if it exists:

(i) $\lim\limits_{x \to 3} \dfrac{\frac{6}{x} - 2}{x - 3} - \lim\limits_{x \to 0} \dfrac{e^{4x} + 1}{e^{4x}}$.

(5 marks)

(ii) $\lim\limits_{x \to -1} \dfrac{\sqrt{x^2 + 15} - 4}{x + 1} + \lim\limits_{x \to \infty} \dfrac{12}{x}$.

(4 marks)

(b) Given that a function, $f(t)$ is written as:

$$f(t) = \begin{cases} \dfrac{t^2 - 16}{t - 4} & ; \quad t < 4 \\ -(t^2) + p & ; \quad 4 \le t < 7 \\ -20 & ; \quad t = 7 \\ 11 - \dfrac{36}{7}t & ; \quad t > 7 \end{cases}.$$

(i) Find the value of p if $\lim_{t \to 4} f(t)$ exist.

(5 marks)

(ii) Determine whether the function is continuous at $t = 7$.

(6 marks)

Q3 (a) Differentiate the following function using implicit differentiation method:

$$x^3 + 2xy + y^2 = 14.$$

(3 marks)

(b) Evaluate the differential if $t^2 = x$ and $t^2 + t = y$ using parametric differentiation.

(3 marks)

(c) The radius of a circular cone with height of 4cm is increasing at the rate of 2cms^{-1}. Calculate the rate of increment for the volume of the cone when the radius is 3cm.

(3 marks)

(d) Given a function as follows:

$$y = \frac{1}{3}x^3 + \frac{1}{2}x^2 - 20x + 6.$$

(i) Determine the local extrema and fill out the **Table Q3(d)(i)**.

(8 marks)

(ii) Hence, sketch the graph.

(3 marks)

Q4 (a) Solve the following integrals:

(i) $$\int_2^3 \left(3s^2 + 3\sqrt[3]{s} - \frac{2}{s^5}\right) ds.$$

(5 marks)

(ii) $\int \frac{\cos(\ln 2x) + x^2 \sin(3x)}{x} dx.$

(8 marks)

(b) Evaluate the following by Simpsons Rule with $h = 0.2$:

$$\int_0^1 \sqrt{x^2 + 1}\, dx.$$

(7 marks)

Q5 (a) (i) In the same Cartesian coordinate, sketch the graphs of $y = \sqrt{x - 4}$ and $y = (x - 4)^2$.

(5 marks)

(ii) Calculate the volume of the solid when the bounded region is rotated about the x-axis by using the cylindrical shells method.

(8 marks)

(b) For the region of $1 \le x \le 4$, evaluate the length of arc of the following curve:

$$y = \frac{4}{3}\left(x^2 + \frac{1}{2}\right)^{\frac{3}{2}}.$$

(7 marks)

– END OF QUESTIONS –

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Table Q3(d)(i)

Critical and inflection points							
Test value							
f' behaviour							
f'' behaviour							

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LIST OF FORMULA

Table 1: Differentiation

$\frac{d}{dx}[c]=0$	$\frac{d}{dx}[\sin u]=\cos u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[x^n]=nx^{n-1}$	$\frac{d}{dx}[\cos u]=-\sin u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[cu]=c\frac{du}{dx}$	$\frac{d}{dx}[\ln\|u\|]=\frac{1}{u}\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[u\pm v]=\frac{du}{dx}\pm\frac{dv}{dx}$	$\frac{d}{dx}\left[e^u\right]=e^u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[uv]=u\frac{dv}{dx}+v\frac{du}{dx}$	$\frac{d}{dx}[a^u]=a^u\ln a\left(\frac{du}{dx}\right)$
$\frac{d}{dx}\left[\frac{u}{v}\right]=\frac{v\frac{du}{dx}-u\frac{dv}{dx}}{v^2}$	$\frac{d}{dx}[\tan u]=\sec^2 u\left(\frac{du}{dx}\right)$
$\frac{d}{dx}[\log_a u]=\frac{1}{u}\log_b e\left(\frac{du}{dx}\right)$	$\frac{d}{dx}[\sec u]=\sec u\tan u\left(\frac{du}{dx}\right)$
Chain Rule: $\frac{dy}{dx}=\frac{dy}{du}.\frac{du}{dx}$	Parametric Differentiation: $\frac{dy}{dx}=\left(\frac{dy}{dt}\Big/\frac{dx}{dt}\right)=\frac{dy}{dt}.\frac{dt}{dx}$

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Table 2: Integration

$\int a\,dx = ax + C$	$\int \sin nx\,dx = -\frac{1}{n}(\cos nx) + C$
$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C, (n \neq -1)$	$\int \cos nx\,dx = \frac{1}{n}(\sin nx) + C$
$\int \frac{1}{nx+b}\,dx = \frac{1}{n}\ln\lvert nx+b\rvert + C$	$\int \tan x\,dx = \ln\lvert \sec x \rvert + C$
$\int \frac{1}{b-nx}\,dx = -\frac{1}{n}\ln\lvert b-nx\rvert + C$	$\int \sec^2 x\,dx = \tan x + C$
$\int e^{nx}\,dx = \frac{1}{n}\left(e^{nx}\right) + C$	$\int \csc^2 x\,dx = -\cot x + C$
$\int e^{nx+b}\,dx = \frac{1}{n}e^{nx+b} + C$	$\int \sec x\,dx = \ln\lvert \sec x + \tan x\rvert + C$
Integration part by part: $\int u\,dv = uv - \int v\,du$	
Improper Integral: $\int_a^{\infty} f(x)dx = \lim_{b\to\infty}\int_a^b f(x)dx$	
Identity: $1 + \tan^2 x = \sec^2 x$	

Area of Region

$$A = \int_a^b [f(x) - g(x)]\,dx \quad \text{or} \quad A = \int_c^d [w(y) - v(y)]\,dy$$

Volume Cylindrical Shells

$$V = \int_a^b 2\pi x\, f(x)\,dx \quad \text{or} \quad V = \int_c^d 2\pi y\, f(y)\,dy$$

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Arc Length

$$L = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx \quad \text{or} \quad L = \int_c^d \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Partial Fraction

$$\frac{a}{x^2 - 1} = \frac{A}{x+1} + \frac{B}{x-1}$$

Simpson's Rule

$$\int_a^b f(x)\, dx \approx \frac{h}{3}\left[\big(f(a) + f(b)\big) + 4\sum_{\substack{i=1 \\ i \text{ odd}}}^{n-1} f(a+ih) + 2\sum_{\substack{i=2 \\ i \text{ even}}}^{n-2} f(a+ih) \right]; \quad n = \frac{b-a}{h}$$

Trapezoidal Rule

$$\int_a^b f(x)\, dx \approx \frac{h}{2}\left[f(a) + f(b) + 2\sum_{i=1}^{n-1} f(a+ih) \right]; \quad n = \frac{b-a}{h}$$