Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER I
## SESSION 2022/2023

| | | |
|---|---|---|
| COURSE NAME | : | ALGEBRA |
| COURSE CODE | : | DAC 11402 |
| PROGRAMME CODE | : | DAA |
| EXAMINATION DATE | : | FEBRUARY 2023 |
| DURATION | : | 2 HOURS AND 30 MINUTES |
| INSTRUCTION | : | 1. ANSWER ALL QUESTIONS<br>2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**<br>3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK |

THIS QUESTION PAPER CONSISTS OF **SEVEN (7)** PAGES

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**Q1** (a) Solve $4^{2\left(x+\frac{1}{2}\right)}-9\left(4^{x}\right)+2=0$.

(5 marks)

(b) Find the values of $x$ that satisfies the equation:

$$\log_2 x+\log_x 2=\frac{10}{3}.$$

(5 marks)

(c) Given that $z_1=5+3i$ and $z_2=4-i$.

(i) Evaluate $\frac{z_1}{z_2}$ in the form $a+bi$.

(3 marks)

(ii) Calculate $z_2 z_1$ in the form $r\left(\cos\theta+i\sin\theta\right)$.

(7 marks)

**Q2** (a) Divide $16x^3+2+11x+20x^2$ by $2x+1$.

(5 marks)

(b) Express the following as a sum of partial fractions:

$$\frac{4x^2+17x+15}{(x+2)^2(x-1)}.$$

(6 marks)

(c) Calculate the root of $f\left(x\right)=4x^2-e^x$ using bisection method. Iterate until $\left|f(c_i)\right|<\varepsilon=0.005$. Show your calculation in three decimal places.

(9 marks)

**Q3** (a) Find $\sum_{k=1}^{7} k\left(2k^2-3k+4\right)$.

(4 marks)

(b) Write down the first fifth terms of the binomial expression of $\frac{1}{\left(1-\frac{3}{2}x\right)^2}$.

(5 marks)



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(c) (i) A circle in **Figure Q3 (c)(i)** is divided into $n$ sectors so that the angles of the sectors form an arithmetic progression. If the angle of the smallest sector is $9°$ and the largest sector is $71°$. Evaluate the value of $n$.

(4 marks)

(ii) The fifth term and the tenth term of geometric series are $\frac{t^4}{162}$ and $\frac{t^9}{39366}$ respectively. Determine the first term, $a$ and the common ratio, $r$.

(7 marks)

**Q4** (a) Find the exact value of $\sin 270°$ by using double-angle formula.

(5 marks)

(b) By using half-angle formula, evaluate the value of $\cos 165°$.

(5 marks)

(c) Prove that $\frac{\cos x}{1-\sin x}+\frac{1-\sin x}{\cos x}=2\sec x$.

(5 marks)

(d) Solve the trigonometric equation $4\sin^2\theta-9\cos\theta=6$ for $0°\leq\theta\leq 360°$.

(5 marks)

**Q5** (a) Given $\mathbf{a}=4\mathbf{i}+3\mathbf{k}$, $\mathbf{b}=-2\mathbf{i}+\mathbf{j}+5\mathbf{k}$ and $\mathbf{c}=8\mathbf{i}+2\mathbf{j}-4\mathbf{k}$.

(i) Evaluate $|\mathbf{a}-2\mathbf{b}+3\mathbf{c}|$.

(4 marks)

(ii) Calculate $2\mathbf{a}\cdot(3\mathbf{b}\times 5\mathbf{c})$.

(6 marks)

(b) Determine the symmetric equation of the line that passes through points $P(0,1,3)$ and $Q(3,1,4)$.

(4 marks)

(c) Given three points $W(-2,4,6)$, $X(0,4,-2)$ and $Y(1,0,-5)$. Find the equation of the plane passing through all the points.

(6 marks)

**– END OF QUESTIONS –**



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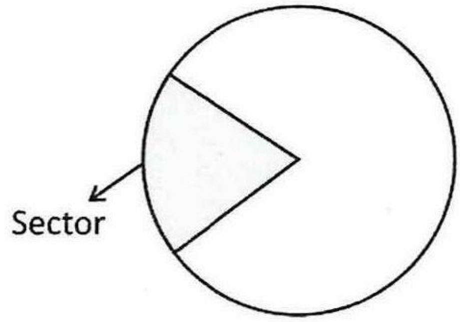


**Figure Q3 (c)(i)**



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**FORMULA**

**Polynomials**

$$\log_a x = \frac{\log_a x}{\log_a b} \qquad x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, \qquad x^2 + bx + c = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c,$$

$$x_{i+2} = \frac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$$

**Partial Fraction**

| |
|---|
| $\frac{P(x)}{(x+b)(x+c)} = \frac{A}{x+b} + \frac{B}{x+c}$ |
| $\frac{P(x)}{x(x+b)(x+c)} = \frac{A}{x} + \frac{B}{x+b} + \frac{C}{x+c}$ |
| $\frac{P(x)}{(x+b)^2} = \frac{A}{x+b} + \frac{B}{(x+b)^2}$ |
| $\frac{P(x)}{(x+b)(x^2+c)} = \frac{A}{(x+b)} + \frac{Bx+C}{(x^2+c)}$ |

**Sequence and Series**

$$\sum_{k=1}^{n} c = cn, \quad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}, \quad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{k=1}^{n} k^3 = \left(\frac{n(n+1)}{2}\right)^2$$

$$x_n = a + (n-1)d \qquad S_n = \frac{n}{2}[2a + (n-1)d], \qquad S_n = \frac{n}{2}(a + u_n)$$

$$x_n = ar^{n-1}, \quad S_n = \frac{a(r^n - 1)}{r-1}, r > 1 \text{ **OR** } S_n = \frac{a(1-r^n)}{1-r}, r < 1, \qquad S_\infty = \frac{a}{1-r}.$$

$$x_n = S_n - S_{n-1}$$

$$(1+b)^n = 1 + nb + \frac{n(n-1)}{2!}b^2 + \frac{n(n-1)(n-2)}{3!}b^3 + \frac{n(n-1)(n-2)(n-3)}{4!}b^4 + \cdots$$



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**FORMULA**

**Trigonometry**

$\sin^2 x + \cos^2 x = 1, \quad \tan^2 x + 1 = \sec^2 x, \quad 1 + \cot^2 x = \csc^2 x$

$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$

$\tan(\alpha \pm \beta) = \dfrac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$

$\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$\sin 2\theta = 2\sin\theta\cos\theta$, $\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta$

$\sin\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$, $\cos\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$, $\tan\dfrac{\theta}{2} = \pm\sqrt{\dfrac{1-\cos\theta}{1+\cos\theta}}$

**Table 1: Vector**

| | |
|---|---|
| $\lvert\mathbf{u}\rvert = \sqrt{a^2+b^2+c^2}$ | $\hat{\mathbf{u}} = \dfrac{\mathbf{u}}{\lvert\mathbf{u}\rvert}$ |
| $\mathbf{u}\bullet\mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$ | $\mathbf{u}\bullet\mathbf{v} = \lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert\cos\theta$ |
| $\theta = \cos^{-1}\left(\dfrac{\mathbf{u}\bullet\mathbf{v}}{\lvert\mathbf{u}\rvert\lvert\mathbf{v}\rvert}\right)$ | $A = \dfrac{1}{2}\lvert\mathbf{u}\times\mathbf{v}\rvert$ |
| $\mathbf{u}\times\mathbf{v} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix}$ | $\dfrac{x-x_0}{a_1} = \dfrac{y-y_0}{a_2} = \dfrac{z-z_0}{a_3}$ |
| $A(x-x_0)+B(y-y_0)+C(z-z_0)=0$ | |



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**Table 2: Complex Number**

| | |
|---|---|
| $z = a + bi$<br>$\bar{z} = a - bi$ | $z = r(\cos\theta + i\sin\theta)$ |
| $r = \sqrt{a^2 + b^2}$ | $\theta = \tan^{-1}\left(\frac{b}{a}\right)$ |
| $z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2)]$ | |
| $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + i\sin(\theta_1 - \theta_2)]$ | |



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