Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	ENGINEERING MATHEMATICS
COURSE CODE	:	DAE 12003
PROGRAMME CODE	:	DAE
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER ALL QUESTIONS. 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) The function $f(x)$ is defined by the **Figure Q1(a)**. Find:

(i) $\lim_{x \to 2^+} f(x)$.

(1 mark)

(ii) $f(2)$.

(1 mark)

(iii) $\lim_{x \to 0} f(x)$.

(2 mark)

(b) Given $f(x) = \dfrac{2}{\sqrt{3} - x}$ and $g(x) = 3 - x^2$. Evaluate $\lim_{x \to \sqrt{3}} \left[f(x) g(x) \right]$.

(4 marks)

(c) Find $\lim_{x \to 5} \dfrac{x - 5}{\sqrt{x^2 + 11} - 6}$.

(5 marks)

(d) The function $f(x)$ is defined as follows:

$$f(x) = \begin{cases} x^2 & ; \quad x \le 1 \\ 4x + b & ; \quad 1 < x < 2. \\ x^3 & ; \quad x \ge 2 \end{cases}$$

(i) Find the value of b such that $\lim_{x \to 1} f(x)$ exist.

(3 marks)

(ii) Determine whether the function $f(x)$ is continuous at $x = 2$. Justify the answer.

(4 marks)

Q2 (a) Given that $y = \ln(3 - \sin x)$. Find and simplify $\dfrac{d^2 y}{dx^2}$. [Hint: $\sin^2 x + \cos^2 x = 1$]

(7 marks)

(b) If $\dfrac{x}{2} + e^{xy^3} = (y - 3)^4 + 3\cos y + 4$. Determine $\dfrac{dy}{dx}$ in terms of x and y by using implicit differentiation.

(9 marks)

(c) By using L'Hopital's rule, evaluate $\lim_{t\to 0}\frac{1-\cos t}{t^2}$.

(4 marks)

Q3 (a) By using integration by part, solve $\int 2xe^{-x}\,dx$.

(4 marks)

(b) Evaluate $\int_0^{\pi} x^2\cos 4x\,dx$ by using tabular method.

(7 marks)

(c) **Figure Q3(c)** shows region ***R*** bounded by the functions $y=4x-x^2$ and $y=8x-2x^2$.

(i) Find the points A and B.

(4 marks)

(ii) Hence, find the volume of the solid obtained when the region ***R*** is revolved about y - axis.

(5 marks)

Q4 (a) Find the Laplace transform and simplify the following functions:

(i) $f(t)=\frac{7}{3}-\frac{2}{5}t^4$.

(2 marks)

(ii) $f(t)=2\cos 8t+3\sin 8t$.

(3 marks)

(iii) $f(t)=2e^{-3t}\sinh 5t$.

(5 marks)

(b) Find the inverse Laplace for the expressions below:

(i) $\frac{7}{s^5}-\frac{8}{s}$.

(5 marks)

(ii) $\frac{s+14}{(s+3)^2}$.

(5 marks)

Q5 Solve the given initial value problem of the differential equations using Laplace transform:

(a) $y'' - 3y' + 2y = 0$ where $y(0) = 1$ and $y'(0) = 0$.

(10 marks)

(b) $y'' + 2y' + 5y = 0$ where $y(0) = 1$ and $y'(0) = 0$

(10 marks)

- END OF QUESTIONS -

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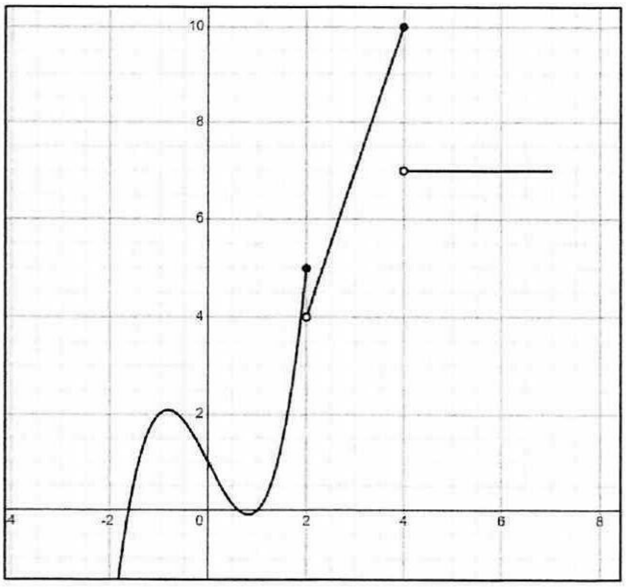

Figure Q1(a)

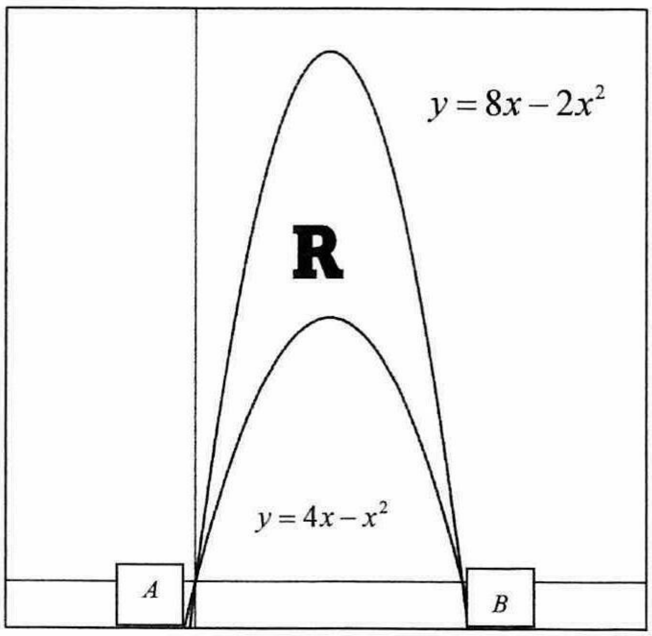

Figure Q3(c)

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Formula

Table 1: Integration and Differentiation

Integration	**Differentiation**
$\int x^n dx = \frac{x^{n+1}}{n+1} + C$	$\frac{d}{dx} x^n = nx^{n-1}$
$\int \frac{1}{x} dx = \ln\lvert x\rvert + C$	$\frac{d}{dx} \ln x = \frac{1}{x}$
$\int \frac{1}{a-bx} dx = -\frac{1}{b} \ln\lvert a-bx\rvert + C$	$\frac{d}{dx} \ln(u) = \frac{1}{u}\frac{du}{dx}$ **where** $u = f(x)$
$\int e^{ax+b} dx = \frac{1}{a} e^{ax+b} + C$	$\frac{d}{dx} e^u = e^u \frac{du}{dx}$
$\int \sin(ax+b)\, dx = -\frac{1}{a}\cos(ax+b) + C$	$\frac{d}{dx} \sin u = \cos u \frac{du}{dx}$
$\int \cos(ax+b) dx = \frac{1}{a}\sin(ax+b) + C$	$\frac{d}{dx} \cos u = -\sin u \frac{du}{dx}$
$\int \sec^2(ax+b)\, dx = \tan(ax+b) + C$	$\frac{d}{dx} \tan u = \sec^2 u \frac{du}{dx}$
$\int \csc^2(ax+b)\, dx = -\cot(ax+b) + C$	$\frac{d}{dx} \cot u = -\csc^2 u \frac{du}{dx}$
$\int u\, dv = uv - \int v du$	$\frac{d}{ds}(uv) = u\frac{dv}{ds} + v\frac{du}{ds}$
$\int_a^b f(x) dx = F(b) - F(a)$	$\frac{d}{ds}\left(\frac{u}{v}\right) = \frac{v\frac{du}{ds} - u\frac{dv}{ds}}{v^2}$
Area of Region $A = \int_a^b [f(x) - g(x)] dx$ **or** $A = \int_c^d [w(y) - v(y)] dy$	Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ Parametric Differentiation: $\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx}$

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Table 2: Partial Fraction

$\frac{a}{(s+b)(s-c)}=\frac{A}{s+b}+\frac{B}{s-c}$
$\frac{a}{s(s-b)(s-c)}=\frac{A}{s}+\frac{B}{s-b}+\frac{C}{s-c}$
$\frac{a}{(s+b)^2}=\frac{A}{s+b}+\frac{B}{(s+b)^2}$
$\frac{a}{(s+b)(s^2+c)}=\frac{A}{(s+b)}+\frac{Bs+C}{(s^2+c)}$

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Table 3: Laplace and Inverse Laplace Transforms

$L\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$	
$f(t)$	$F(s)$
k	$\frac{k}{s}$
$t^n, n = 1, 2, \ldots$	$\frac{n!}{s^{n+1}}$
e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$
$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$
$\cosh at$	$\frac{s}{s^2-a^2}$
First Shift Theorem	
$e^{at}f(t)$	$F(s-a)$
Multiply with t^n	
$t^n f(t), n = 1, 2, \ldots$	$(-1)^n \frac{d^n F(s)}{ds^n}$
Initial Value Problem	
$L\{y(t)\} = Y(s)$	
$L\{y'(t)\} = sY(s) - y(0)$	
$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$	