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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2022/2023

COURSE NAME	:	ALGORITHMS AND COMPLEXITIES
COURSE CODE	:	BIE 20303
PROGRAMME CODE	:	BIP
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWERS **ALL** QUESTIONS. 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**. 3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

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Q1 **Figure Q1** shows the Dijkstra's algorithm for finding the length of a shortest path between two vertices in a weighted simple connected graph. Extend it so that a shortest path between these vertices is constructed.

procedure *Dijkstra*(G: weighted connected simple graph, with
    all weights positive)
{G has vertices $a = v_0, v_1, \ldots, v_n = z$ and lengths $w(v_i, v_j)$
    where $w(v_i, v_j) = \infty$ if $\{v_i, v_j\}$ is not an edge in G}
for $i := 1$ **to** n
    $L(v_i) := \infty$
$L(a) := 0$
$S := \emptyset$
{the labels are now initialized so that the label of a is 0 and all
    other labels are ∞, and S is the empty set}
while $z \notin S$
    $u :=$ a vertex not in S with $L(u)$ minimal
    $S := S \cup \{u\}$
    for all vertices v not in S
        if $L(u) + w(u, v) < L(v)$ **then** $L(v) := L(u) + w(u, v)$
        {this adds a vertex to S with minimal label and updates the
        labels of vertices not in S}
return $L(z)$ {$L(z) =$ length of a shortest path from a to z}

Figure Q1

(20 marks)

Q2 Using **Figure Q2**, show that the Dijkstra's algorithm may not work if edges can have negative weights.

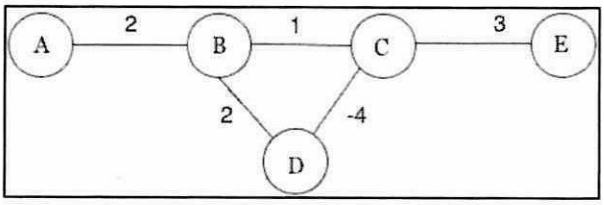

Figure Q2

(20 marks)

Q3 The roads represented by the graph in **Figure Q3.2** are all unpaved. The lengths of the roads between pairs of towns are represented by edge weights. Use Kruskal's algorithm in **Figure Q3.1** to find which roads should be paved so that there is a path of paved roads between each pair of towns so that a minimum road length is paved.

procedure *Kruskal*(G: weighted connected undirected graph with n vertices)
T := empty graph
for $i := 1$ **to** $n - 1$
  e := any edge in G with smallest weight that does not form a simple circuit when added to T
  $T := T$ with e added
return T {T is a minimum spanning tree of G}

Figure Q3.1

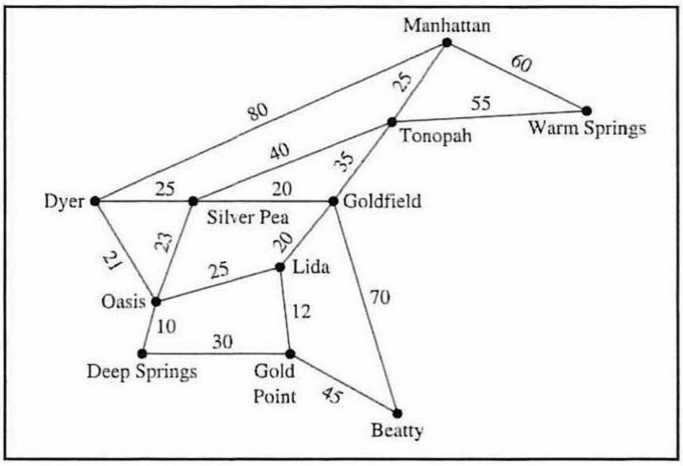

Figure Q3.2

(20 marks)

Q4 A maximum spanning tree of a connected weighted undirected graph is a spanning tree with the largest possible weight. Devise an algorithm similar to Kruskal's algorithm in **Figure Q3.1** for constructing a maximum spanning tree of a connected weighted graph.

(20 marks)

Q5 A minimum spanning forest in a weighted graph is a spanning forest with minimal weight. Explain how Prim's algorithm, which is represented in **Figure Q5** and Kruskal's algorithm in **Figure Q3.1** can be adapted to construct minimum spanning forests.

procedure *Prim*(G: weighted connected undirected graph with n vertices)
T := a minimum-weight edge
for $i := 1$ **to** $n - 2$
  e := an edge of minimum weight incident to a vertex in T and not forming a simple circuit in T if added to T
  $T := T$ with e added
return T {T is a minimum spanning tree of G}

Figure Q5

(20 marks)

-END OF QUESTIONS –