Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER 1
SESSION 2022/2023

COURSE NAME	:	ALGEBRA
COURSE CODE	:	BIC10303
PROGRAMME CODE	:	BIS/BIP/BIW/BIM
EXAMINATION DATE	:	FEBRUARY 2023
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER ALL QUESTIONS.
		2. THIS FINAL EXAMINATION IS CONDUCTED VIA **CLOSED BOOK**.
		3. STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK.

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

Q1 (a) Solve each of the inequalities below. Write the solution in interval notation.

(i) $\frac{3x^2+2x-8}{2x-1} \leq 0.$

(5 marks)

(ii) $5|2x-3| \geq 4|x-5|.$

(8 marks)

(b) Answer **Q1b(i) – (ii)** based on the case study below:

> An architect is designing the entryway of a restaurant. He wants to put a triangular window above the doorway. Due to energy restrictions, the window can only have an area of 120 meter square feet and the architect wants the base to be 4 feet more than twice the height.

Find the height and the base of the window.

(5 marks)

Q2 (a) For each of the following sequence, write a formula for the *n*th term, write the sum of the sequence in sigma notation and then find the summation.

(i) $2, 6, 18, 54, \ldots$

(3 marks)

(ii) $5, 2, \frac{4}{5}, \frac{8}{25}, \ldots$

(3 marks)

(b) Kumon Institute is an educational center which used a special technique to teach reading primarily for young students. Using the technique, students will gradually be instructed to increase their daily reading. On the first day of January, an English teacher instructs his students to read five pages of a novel and everyday thereafter increase their daily reading by two pages. If his students follow the instruction, then

(i) how many pages students should read on the last day of the second week of January.

(3 marks)

(ii) how many pages will they read during January.

(3 marks)

(iii) if the monthly target is to read the minimum of thrice the total pages of the preceding month and this technique can only be terminated once the monthly reading reaches the minimum of 20000 pages per month. In which month this technique will end? Show your calculation with the nth term formula.

(5 marks)

Q3 (a) Given that

$$Q = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 2 & 0 & 1 & 4 \end{bmatrix}.$$

Find the inverse of matrix Q by using the elementary row operation method.

(5 marks)

(b) Find the determinant of matrix R if given

$$R = \begin{bmatrix} 1 & 0 & 4 & -6 \\ 2 & 5 & 0 & 3 \\ -1 & 2 & 3 & 5 \\ 2 & 1 & -2 & 2 \end{bmatrix}.$$

(7 marks)

(c) Compute the eigenvalues and eigenvectors of the following matrix.

$$P = \begin{bmatrix} 5 & -3 \\ -6 & 2 \end{bmatrix}.$$

(8 marks)

Q4 (a) Solve the following system of equations by Gauss-Elimination method.

$$\begin{aligned} 28r + 4s - t &= 32 \\ r + 3s + 10t &= 24 \\ 2r + 17s + 4t &= 35 \end{aligned}$$

(8 marks)

(b) A biologist has placed three strains of bacteria denoted I, II and III in a test tube, where they will feed on three different food sources A, B and C. Each day 700 units of A, 400 units of B and 500 units of C are placed in the test tube. Each bacteria consumes a certain number of units of each food per day as shown in **Table Q4** below.

Table Q4: Bacteria food consumption per day

	Bacteria Strain I	Bacteria Strain II	Bacteria Strain III
Food A	0	1	2
Food B	5	1	0
Food C	1	3	1

(i) Obtain a system of linear equations based on above problem.

(2 marks)

(ii) Determine the number of bacteria of each strain that can coexist in the test tube by using Cramer's rule method.

(10 marks)

Q5 (a) Graph the system of inequalities below.

$$x \geq 0\,,\ y \geq 0\,,$$
$$x + 3y \leq 4,$$
$$2x + 3y \leq 5.$$

(5 marks)

(b) Maximize and minimize the value of $\boldsymbol{D} = 3\boldsymbol{x} + 4\boldsymbol{y}$ subject to the constraints

$$\boldsymbol{x} + 2\boldsymbol{y} \leq 14$$
$$3\boldsymbol{x} - \boldsymbol{y} \geq 0$$
$$\boldsymbol{x} - \boldsymbol{y} \geq 2$$

(8 marks)

(c) A ski manufacturer makes two types of skis, downhill and cross-country. Using the information in **Table Q5** below, how many of each type of ski should be made for a maximum profit to be archived? What is the maximum profit?

Table Q5: Hour's consumption and Profit for each type of ski.

	Downhill	Cross-Country	Maximum Time Available
Manufacturing Time per Ski	2 hours	1 hour	40 hours
Finishing Time per Ski	1 hour	1 hour	32 hours
Profit per Ski	RM 70	RM 50	

(12 marks)

END OF QUESTIONS