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**UNIVERSITI TUN HUSSEIN ONN MALAYSIA**

**FINAL EXAMINATION  
SEMESTER I  
SESSION 2021/2022**

COURSE NAME : PARTIAL DIFFERENTIAL EQUATIONS

COURSE CODE : BWA 30303

PROGRAMME CODE : BWA

EXAMINATION DATE : JANUARY / FEBRUARY 2022

DURATION : 3 HOURS

INSTRUCTION : 1. ANSWER ALL QUESTIONS.

2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK**.

THIS QUESTION PAPER CONSISTS OF **THREE (3)** PAGES

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**TERBUKA**

- Q1** Let  $f, g, h$  be functions  $\mathbb{R}^2 \rightarrow \mathbb{R}$  with continuous second order partial derivatives. If  $w = f(x, y)$ ,  $x = g(u, v)$  and  $y = h(u, v)$ , find the expression for  $w_{uu}$  in terms of the partial derivatives of  $f, g$  and  $h$ .

(10 marks)

- Q2** By using the coordinate method solve  $4 \frac{\partial f}{\partial x} + 3 \frac{\partial f}{\partial y} + 2f = 1$ .

[Hint: use integrating factor to solve the first-order ordinary differential equation]

(10 marks)

- Q3** By using the linear method, show that the general solution of  $u_{yy} - u_y = x^2$  is given by  $u(x, y) = -x^2 y + e^y f(x) + g(x)$ .

(10 marks)

- Q4** Consider the partial differential equation

$$\frac{\partial^2 f}{\partial x^2} - 4 \frac{\partial^2 f}{\partial y^2} + 3 \frac{\partial f}{\partial x} + f = 0.$$

Identify the classification of the equation and predict its canonical form. Using transformation  $f(x, y) = z(x, y)e^{px}$  and rescaling independent variables  $x = \alpha\xi$  and  $y = \beta\eta$ , show that the correspond canonical equation is given by  $\frac{\partial^2 z}{\partial \xi^2} - \frac{\partial^2 z}{\partial \eta^2} - z = 0$  when  $p = -\frac{3}{2}$ ,  $\alpha = \frac{2}{\sqrt{5}}$  and

$$\beta = \frac{4}{\sqrt{5}}.$$

(18 marks)

**Q5** Let

$$f(x) = x \text{ over interval } -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}.$$

- (a) Sketch a graph of  $f(x)$  and identify the period in the range  $-2\pi < x < 2\pi$ . Indicate the period in the graph. Determine whether  $f(x)$  is even or odd function. State the symmetrical axis or point of  $f(x)$  from the graph.

(5 marks)

- (b) Find the Fourier series in the range  $-\frac{\pi}{2} < x < \frac{\pi}{2}$  for the function  $f$  up to three terms.

(13 marks)

- (c) Find the sum of the Fourier series when  $x = \frac{\pi}{4}$ .

(2 marks)

**Q6** Consider the diffusion temperature  $\theta(x,t)$  in a homogeneous one-dimensional heat conducting rod of length  $L$ . The initial temperature at time  $t=0$  is given by  $\sin x$  and two ends are fixed at zero temperature. Assume that there is no internal source that generating heat to the system and the thermal diffusivity parameter,  $\alpha=1$ .

- (a) Sketch the geometrical configuration of the system and write the corresponds governing equation including the Dirichlet boundary conditions.

(6 marks)

- (b) Solve the problem using separation of variables.

(26 marks)

**-END OF QUESTIONS -**