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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	OPTIMAL CONTROL
COURSE CODE	:	BWA 31303
PROGRAMME CODE	:	BWA
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK**

THIS QUESTION PAPER CONSISTS OF **FOUR (4)** PAGES

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Q1 (a) Consider the Hamiltonian function given by

$$H(\mathbf{x},\mathbf{u},\boldsymbol{\lambda},t) = L(\mathbf{x},\mathbf{u},t) + \boldsymbol{\lambda}^T f(\mathbf{x},\mathbf{u},t),$$

where $L:\Re^n \times \Re^m \times \Re \to \Re$ and $f:\Re^n \times \Re^m \times \Re \to \Re^n$.

(i) Explain the reason of using the Hamiltonian function in handling the control system.
(ii) Decompose the Hamiltonian function to the state equation and the performance index.
(iii) Describe the optimality conditions that are derived from the Hamiltonian function.

(15 marks)

(b) Given that

$$L = 2x_1^2 + x_2^2 + u^2 \text{ and } f = \begin{pmatrix} 2x_1 - x_2 + u \\ x_1 + x_2 \end{pmatrix}.$$

(i) Justify the Riccati equation.
(ii) Deduce the optimal control function.

(10 marks)

Q2 (a) Consider the control system

$$\dot{x} = 2x + u,$$

which minimizes the performance index

$$J = \frac{1}{2}\int_0^1 (2x^2 + u^2)\ dt.$$

(i) Construct an augmented performance index.
(ii) Judge the optimality conditions by using the variational approach given that $\lambda(0) = \lambda(1) = 0$.

(15 marks)

(b) Consider the performance index

$$J(t_0) = \frac{1}{2}S(T)(x(T))^2 + \frac{1}{2}\int_{t_0}^{T} (x^2 + u^2)\ dt$$

with $\dot{x} = x + u$ and $u = -Kx$, where $x \in \Re$ and $u \in \Re$.

(i) Summarize that the cost to go on any interval $[t, T]$ is given by

$$J(t) = \frac{1}{2}S(t)(x(t))^2.$$

(ii) State any additional information that is needed for **Q2(b)(i)**.

(10 marks)

Q3 (a) Consider the state and costate equations

$$\dot{\mathbf{x}} = \begin{pmatrix} 0 & 1 \\ 5 & 2 \end{pmatrix}\mathbf{x} - \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\boldsymbol{\lambda},$$

$$-\dot{\boldsymbol{\lambda}} = \begin{pmatrix} 3 & 0 \\ 0 & 2 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 0 & 5 \\ 1 & 2 \end{pmatrix}\boldsymbol{\lambda}.$$

(i) Define the homogeneous Hamiltonian system.

(ii) Determine the eigenvalues of the Hamiltonian matrix.

(14 marks)

(b) The Hamiltonian matrix

$$H = \begin{pmatrix} 2 & -1 \\ -3 & -2 \end{pmatrix}$$

has eigenvalues $\mu_1 = \sqrt{7}$ and $\mu_2 = -\sqrt{7}$. With knowing the eigenvectors $\mathbf{v}_1$ and $\mathbf{v}_2$, an analytic solution to the Riccati equation can be given by

$$S(t) = (w_{21} + w_{22}V(t))(w_{11} + w_{12}V(t))^{-1},$$

where

$$W = \begin{pmatrix} 1 & 1 \\ 2-\sqrt{7} & 2+\sqrt{7} \end{pmatrix}$$

is the matrix of eigenvectors. Conclude the analytic solution to the Riccati equation given that

$$S(T) = 0,\ V(T) = -w_{22}^{-1}w_{21} \text{ and } V(t) = e^{-M(T-t)}V(T)e^{-M(T-t)}.$$

(11 marks)

Q4 Consider the control system

$$x(k+1) = 3x(k) + u(k),$$

with $x(0) = 1$, which minimize the performance index

$$J = \frac{1}{2}x(5)^2 + \frac{1}{2}\sum_{k=0}^{4}(10x^2 + u^2).$$

(a) Express the Joseph stabilized version of the Riccati equation.

(3 marks)

(b) Calculate the sequences of the Kalman feedback gain.

(11 marks)

(c) Compute the feedback control sequences.

(8 marks)

(d) Predict the minimum value of the performance index.

(3 marks)

– END OF QUESTIONS –