CONFIDENTIAL

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022

COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL
COURSE CODE	:	BDU 21403
PROGRAMME CODE	:	BDC / BDM
EXAMINATION DATE	:	JULY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **FOUR (4)** QUESTIONS **ONLY.** 2. THIS FINAL EXAMINATION IS CONDUCTED VIA **OPEN BOOK**.

THIS QUESTION PAPER CONSISTS OF **NINE (9)** PAGES

CONFIDENTIAL

Q1 An autopilot that controls an aircraft in the roll axis only is also known as wing levelers, reflecting their single capability. Consider a roll autopilot of a jet fighter shown in **Figure Q1(a).**

(a) Determine the closed-loop transfer function $\phi(s)/\phi_d(s)$ if $K_g = 1.5$.

(5 marks)

(b) Determine the roots of the characteristic equations if the controller gain is set at $K = 1$, 5 and 9.

(3 marks)

(c) Using the concept of dominant roots, differentiate the effect of gain selection (i.e., $K = 1$, 5 and 9) towards the closed-loop system's time response. Suggest the range of gain values that will make the control system unstable.

(9 marks)

(d) Suggest a suitable controller gain, *K*, to enable the jet fighter's roll response to exhibit percentage overshoot (OS%) equals to 16%. Calculate the resulting peak time.

(8 marks)

Q2 (a) The short-period response characteristics of an aircraft are of particular importance in flying and handling quality. The reduced-order state-space model corresponding to short-period mode approximation for a fixed-wing UAV aircraft is given as follows:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_w & Z_q \\ M_w & M_q \end{bmatrix} \begin{bmatrix} w \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_e} \\ M_{\delta_e} \end{bmatrix} \delta_e$$

with the following stability derivatives:

$$Z_w = -6.6 \qquad M_q = -12.5$$
$$Z_q = 16.7 \qquad Z_{\delta_e} = -3.0$$
$$M_w = -6.5 \qquad M_{\delta_e} = -133.7$$

Find the solution to the state-space model using Paynter's numerical method. Use time interval, $\Delta t = 0.01$ to solve the numerical problem.

(6 marks)

(b) If the input of the system, u_1 is applied with 1° elevator step input with output equation and initial condition given as follows:

$$q_k = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} w_k \\ q_k \end{bmatrix}$$
$$\begin{bmatrix} w_0 \\ q_0 \end{bmatrix} = \begin{bmatrix} 1.5 \\ 0.15 \end{bmatrix}$$

Determine the output response, q_k of the state equation for three (3) iterations.

(6 marks)

(c) Examine the stability derivatives' influence on the damping ratio and natural frequency of the short period motion.

(5 marks)

(d) Comment on the time response characteristics of the short period motion obtained for this aircraft. Do your findings agree with the handling quality criteria shown in **Figure Q2(d)**?

(8 marks)

Q3 Consider a pitch control system as shown in **Figure Q3,** with transfer functions for each component in the control system are given as:

$$K(s) = K_P + \frac{K_I}{s} + K_D s$$

$$G_?(s) = \frac{6.5}{(s+10)(s^2+0.3s+2.2)}$$

(a) Consider the controller's transfer function is set with $K(s) = K_P$, examine the closed-loop pole movement of the pitch control system if K_P varies from 0 to ∞. Discuss whether the Ziegler and Nichols tuning method is a suitable tuning method to be used with the pitch control system. Determine the damped frequency, ω_d and gain, K, values at the imaginary axis crossing if such a situation exists.

(10 marks)

(b) Develop the automatic controllers (i.e., P, PD, and PID control) for the dynamic system under consideration using the Ziegler and Nichols tuning method. Compare the steady-state error performance of the compensated systems (i.e., P, PD, and PID control). Describe any problems with your design.

(15 marks)

Q4 (a) Describe the physical characteristics of Dutch Roll stability mode.

(3 marks)

(b) The Dutch Roll motion can be approximated using the following equation:

$$\begin{bmatrix} \Delta\dot{\beta} \\ \Delta\dot{r} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{u_0} & -\left(1-\frac{Y_r}{u_0}\right) \\ N_\beta & N_r \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta r \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta r}}{u_0} \\ N_{\delta r} \end{bmatrix} \delta r$$

Assume the aircraft has the following stability derivative characteristics as follows:

$Y_\beta = -7.5$ ft/s^2 $\quad$ $Y_r = 3.1$ ft/s

$N_\beta = 3.9$ s^{-2} $\quad$ $N_r = -0.35$ s^{-1}

$Y_{\delta r} = -5.9$ ft/s^2 $\quad$ $N_{\delta r} = 0.515$ s^{-2}

$u_0 = 120$ ft/s

(i) Determine the characteristic equation of the Dutch Roll mode.

(5 marks)

(ii) Determine the eigenvalues of the Dutch Roll mode.

(2 marks)

(iii) Determine the damping ratio, natural frequency, period, time to half amplitude, and the number of cycles to half amplitude for the Dutch Roll mode.

(5 marks)

(c) A rotorcraft based unmanned aerial vehicle (UAV) is installed in a test rig to test the functionality of the autopilot system. The UAV prototype is constrained so that it can only rotate about the z-axis (i.e., producing a pure yawing motion). The yaw angle to the rudder input transfer function can be modeled according to:

$$\frac{\psi(s)}{\delta_{rud}(s)} = \frac{10}{s^2 + 0.5s + 2}$$

Design a heading control system so that the model has the system can exhibit closed-loop performance with damping ratio, $\xi = 0.6$ and setting time, $t_s \leq 2.5$ s. Consider the sensor used in the control system design to be a perfect device.

(10 marks)

Q5 (a) Explain how the root locus plot can be used to evaluate the effect of feedback on the characteristic of motion?

(2 marks)

(b) An attitude control system for a satellite vehicle within the earth's atmosphere is shown in **Figure Q5(b)**. The transfer functions of the system are given as follows:

$$G(s) = \frac{K(s + 0.2)}{(s + 0.9)(s - 0.6)(s - 0.1)}$$

$$G_c(s) = \frac{(s^2 + 4s + 6.25)}{(s + 4)}$$

Suggest a range of gain, *K,* that results in a system with a settling time of less than 10 s and a damping ratio for the complex roots greater than 0.643. Provide a detailed root locus plot for the closed-loop system as *K* varies from 0 to ∞ with necessary calculation such as the asymptote angle, centroid, break-in/out, angle of departure/arrival or imaginary axis intersection point to support your answer.

(23 marks)

-END OF QUESTION-

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM II / 2021/2022	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

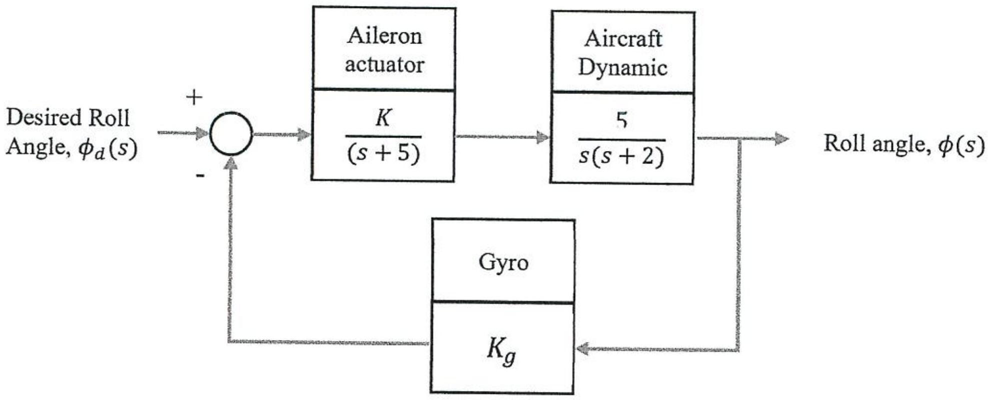

Figure Q1(a) Roll angle control system.

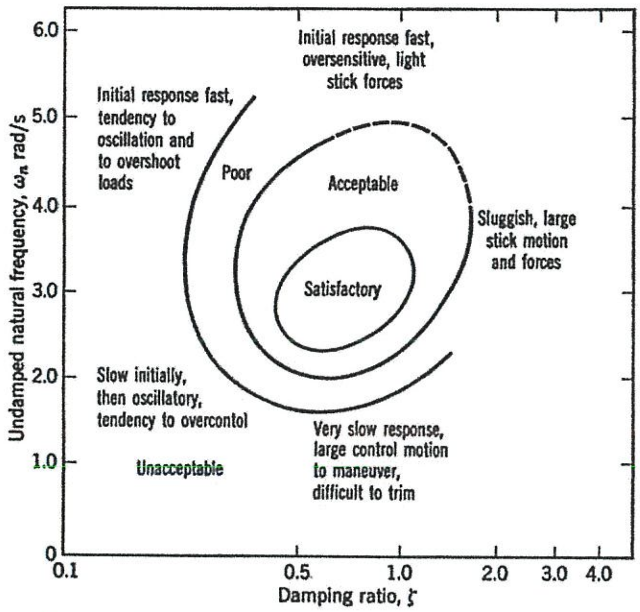

Figure Q2(d) The short period flying quality.

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM II / 2021/2022	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

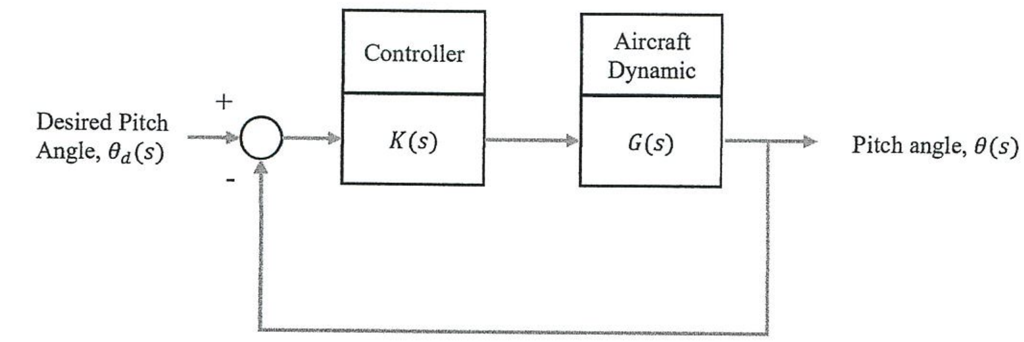

Figure Q3 Simplified block diagram for pitch angle control system.

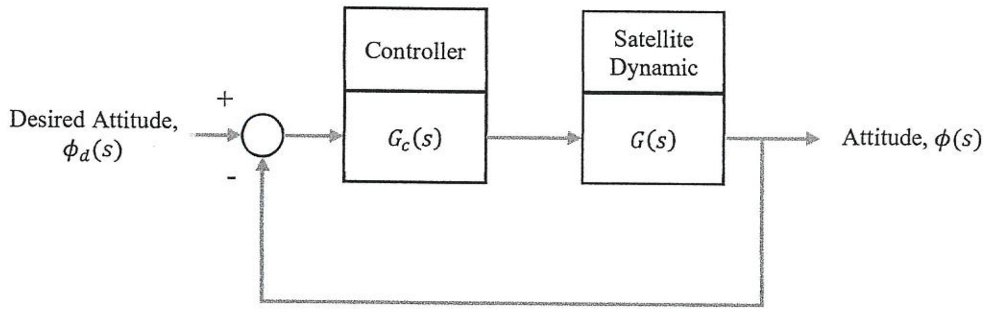

Figure Q5(b) The block diagram for the satellite control system.

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM II / 2021/2022	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

Key Equations

The relevant equations used in this examination are given as follows:

1. The determinant of a 3×3 matrix:

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix} \quad (1)$$

2. Partial fraction for $F(s)$ with real and distinct roots in the denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2}{(s+p_2)} + \cdots + \frac{K_m}{(s+p_m)} \quad (2)$$

3. Partial fraction for $F(s)$ with complex or imaginary roots in the denominator:

$$F(s) = \frac{K_1}{(s+p_1)} + \frac{K_2 s + K_3}{(s^2+as+b)} + \cdots \quad (3)$$

4. General first-order transfer function:

$$G(s) = \frac{K}{s+a} \quad (4)$$

5. General second-order transfer function:

$$G(s) = \frac{K}{s^2 + 2\xi\omega_n s + \omega_n} \quad (5)$$

6. The closed-loop transfer function:

$$T(s) = \frac{G(s)}{1+G(s)H(s)} \quad (6)$$

where $G(s)$ is the transfer function of the open-loop system, and $H(s)$ is the transfer function in the feedback loop.

7. The final value theorem:

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} sY(s) \quad (7)$$

8. Time response:

$$T_r = \frac{2.2}{a} \quad (8)$$

$$T_s = \frac{4}{a} \quad (9)$$

$$\%OS = e^{-\left(\frac{\xi\pi}{\sqrt{1-\xi^2}}\right)} \times 100\% \quad (10)$$

$$\xi = \frac{-\ln\left(\%\frac{OS}{100}\right)}{\sqrt{\pi^2 + \left(\ln\left(\%\frac{OS}{100}\right)\right)^2}} \quad (11)$$

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM II / 2021/2022	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU 21403

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}} = \frac{\pi}{\omega_d} \quad (12)$$

$$T_s = \frac{4}{\xi\omega_n} = \frac{4}{\eta} \quad (13)$$

$$P = \frac{2\pi}{\omega_d} \quad (14)$$

$$t_{1/2} = \frac{0.693}{|\eta|} \quad (15)$$

$$N_{1/2} = 0.110\frac{|\omega_d|}{|\eta|} \quad (16)$$

9. Estimation of parameter q (Paynter Numerical Method)

$$q = \max\left|A_{ij}\Delta t\right| \quad (17)$$

10. Estimation of the integer value of p (Paynter Numerical Method)

$$\frac{1}{p!}(nq)^p e^{nq} \leq 0.001 \quad (18)$$

11. Numerical solution of state equation:

$$\mathbf{x}_{k+1} = \boldsymbol{M}\mathbf{x}_k + \boldsymbol{N}\boldsymbol{\eta}_k$$

with matrix $\boldsymbol{M}$ and N are given by the following matrix expansion:

$$\boldsymbol{M} = e^{A\Delta t} = \boldsymbol{I} + A\Delta t + \frac{1}{2!}\boldsymbol{A}^2\Delta t^2 \ldots \quad (19)$$

$$\boldsymbol{N} = \Delta t\left(\boldsymbol{I} + \frac{1}{2!}\boldsymbol{A}\Delta t + \frac{1}{3!}\boldsymbol{A}^2\Delta t^2 + \cdots\right)\boldsymbol{B}$$

12. Characteristic equation of the closed loop system:

$$1 + KG(s)H(s) = 0 \quad (20)$$

13. Asymptotes: angle and real-axis intercept:

$$\sigma = \frac{[\sum Real\ parts\ of\ the\ poles - \sum Real\ parts\ of\ the\ zeros]}{n-m} \quad (21)$$

$$\phi_a = \frac{180°[2q+1]}{n-m} \quad (22)$$

14. The solution to determine real axis break-in and breakaway points:

$$\frac{dK(\sigma)}{d\sigma} = 0 \quad (23)$$

FINAL EXAMINATION

SEMESTER/SESSION	:	SEM II / 2021/2022	PROGRAMME CODE	:	BDC/BDM
COURSE NAME	:	AIRCRAFT STABILITY AND CONTROL	COURSE CODE	:	BDU21403

15. An alternative solution to find real axis break-in and breakaway points:

$$\sum \frac{1}{\sigma + z_i} = \sum \frac{1}{\sigma + p_i} \quad (24)$$

16. The angle of departure of the root locus from a pole of $G(s)H(s)$:

$$\theta = 180° + \sum (angles\ to\ zeros) - \sum (angles\ to\ poles) \quad (25)$$

17. The angle of arrival at a zero:

$$\theta = 180° - \sum (angles\ to\ zeros) + \sum (angles\ to\ poles) \quad (26)$$

18. The steady-state error:

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) \quad (27)$$

where the error signal is given as:

$$E(s) = \frac{1}{1 + K(s)G(s)H(s)} \times U(s) \quad (28)$$

19. The characteristic equation for the standard form of the second-order differential equation:

$$\lambda^2 + 2\xi\omega_n\lambda + \omega_n^2 = 0$$

The roots of the characteristic equation are: (29)

$$\lambda_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{1 - \xi^2} \cdot i$$

$$\lambda_{1,2} = \sigma \pm \omega_d$$

20. The calculation of controller gains using the Ziegler-Nichols method: (30)

Table 1 The Ziegler-Nichols tuning method.

Control Type	K_p	K_I	K_D
P	$0.5K_u$	-	-
PI	$0.45K_u$	$1.2\ K_p/T_u$	-
PD	$0.8K_u$	-	$K_P T_u/8$
Classic PID	$0.6K_u$	$2\ K_p/T_u$	$K_P T_u/8$
Pessen Integral Rule	$0.7K_u$	$2.5\ K_p/T_u$	$3K_P T_u/20$
Some Overshoot	$0.33K_u$	$2\ K_p/T_u$	$K_P T_u/3$
No Overshoot	$0.2K_u$	$2\ K_p/T_u$	$K_P T_u/3$

21. Conversion from the state-space model to transfer function model: (31)

$$G(s) = \boldsymbol{C}\frac{adj(s\boldsymbol{I} - \boldsymbol{A})}{\det(s\boldsymbol{I} - \boldsymbol{A})}\boldsymbol{B}$$