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Universiti Tun Hussein Onn Malaysia

# UNIVERSITI TUN HUSSEIN ONN MALAYSIA

## FINAL EXAMINATION
## SEMESTER II
## SESSION 2021/2022

| | | |
|---|---|---|
| COURSE NAME | : | ENGINEERING MATHEMATICS II |
| COURSE CODE | : | BDA 14103 |
| PROGRAMME CODE | : | BDD |
| EXAMINATION DATE | : | JULY 2022 |
| DURATION | : | 3 HOURS |
| INSTRUCTIONS | : | 1. ANSWER **ONLY THREE (3)** FROM **FOUR (4)** QUESTIONS IN **PART A** AND **ALL** QUESTIONS IN **PART B**<br>2. THIS FINAL EXAMINATION IS CONDUCTED **ONLINE** AND IS AN **OPEN-BOOK** ASSESSMENT |

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

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**PART A:** ANSWER ONLY **THREE (3)** FROM **FOUR (4)** QUESTIONS

**Q1** Pak Hairul has a *Musang King* durian farm at the top of Jintan hill. During the dry season, Pak Hairul had to water his durian trees manually. However, he could not water his durian tree during the Hari Raya holidays. With the help of his wife Mak Yan, Pak Hairul has installed a durian tree watering system as shown in the **Figure Q1**. The watering system is a cylindrical tank with 2 m in height and 1 m in diameter. A circular outflow hole is drilled at the bottom of the tank. Determine the height of the water column that needs to be filled in the barrel so that the remaining height is 5% of what has been filled by Pak Hairul after 2 days?

Notes:

1. The volume $dV$ of the outflow during a short period of time $dt$ is $dV = Avdt$ where $A$ is the area of outflow hole
2. The change of volume $dV_{tank}$ of the water in tank is $dV_{tank} = -Bdh$ where $B$ is the cross-sectional area of tank
3. From continuity equation $dV = dV_{tank}$
4. Assume the gravitational acceleration to be 9.80 $m/s^2$ and the outlet hole diameter to be 2 mm.

(20 marks)

**Q2** (a) A mass-spring system shown in **Figure Q2** has a spring coefficient of $k - 6$ N/mm and a damping coefficient of $c$ = 0.5 Ns/mm. The mass is $m$ = 0.5 kg. A continuous force $F(t) = 3e^{3t}$ is exerted on the mass to induce oscillation. The system is modeled by the following differential equation:

$$y'' + \left(\frac{c}{m}\right)y' - \left(\frac{k}{m}\right)y = F(t)$$

Using the method of undetermined coefficient, find the displacement of the mass with respect to time, y(t).

(10 marks)

(b) Solve the following second order differential equation using the method of variation of parameters:

$$y'' - 16y = \frac{x}{e^{4x}}$$

(10 marks)

**Q3** Sketch the following functions and find the Laplace transforms

(a)

$$f(t) = \begin{cases} t^2, & 0 \le t < 1 \\ t, & 1 \le t < 2 \\ 1, & t \ge 2 \end{cases}$$

(10 marks)

(b)

$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ t-1, & 1 \le t < 2 \\ 3-t, & 2 \le t < 3 \\ 0, & t \ge 3 \end{cases}$$

(10 marks)

**Q4** Solve the following initial value differential equation using Laplace transform:

$$y'' + 4y' + 3y = 2e^{-2t} \qquad y(0) = 1, y'(0) = 2$$

(20 marks)

**PART B**: ANSWER **ALL** QUESTIONS

**Q5** Let $f(x)$ be a function of period $2\pi$ such that:

$$f(x) = \begin{cases} x, & 0 < x < \pi \\ \pi, & \pi < x < 2\pi \end{cases}$$

(a) Sketch the graph of $f(x)$ in the interval $0 < x < 4\pi$

(2 marks)

(b) Prove that the Fourier series for $f(x)$ in the interval $0 < x < 2\pi$ is:

$$f(x) = \frac{3\pi}{4} - \frac{2}{\pi}\left[\cos x + \frac{1}{9}\cos 3x + \frac{1}{25}\cos 5x + \cdots\right] - \left[\sin x + \frac{1}{2}\sin 2x + \frac{1}{3}\sin 3x + \cdots\right]$$

(12 marks)

(c) By giving an appropriate value $x$, demonstrate that

$$\frac{\pi}{4} = -1 + \frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \cdots$$

(6 marks)

**Q6** A 3 cm length silver bar with constant cross section area of 1 cm$^2$, density of 10 g/cm$^3$, thermal conductivity of 3 cal/cm.sec.°C and specific heat of 0.15 cal.g.°C, is perfectly insulated laterally with ends kept at temperature 0 °C and initial uniform temperature $f(x) = 25$ °C. The heat equation is:

$$\frac{\partial^2 u}{\partial x^{2x}} = \frac{1}{c^2}\frac{\partial u}{\partial t}$$

where $u$ is temperature and $t$ is time.

(a) Show that $c^2 = 2$

(2 marks)

(b) By using the method of separation of variable, and applying the boundary condition, prove that:

$$u(x,t) = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{3} e^{-\frac{2n^2\pi^2 t}{9}}$$

where $b_n$ is an arbitrary constant.

(12 marks)

(c) By applying the initial condition, find the value of $b_n$

(6 marks)

**- END OF QUESTIONS –**

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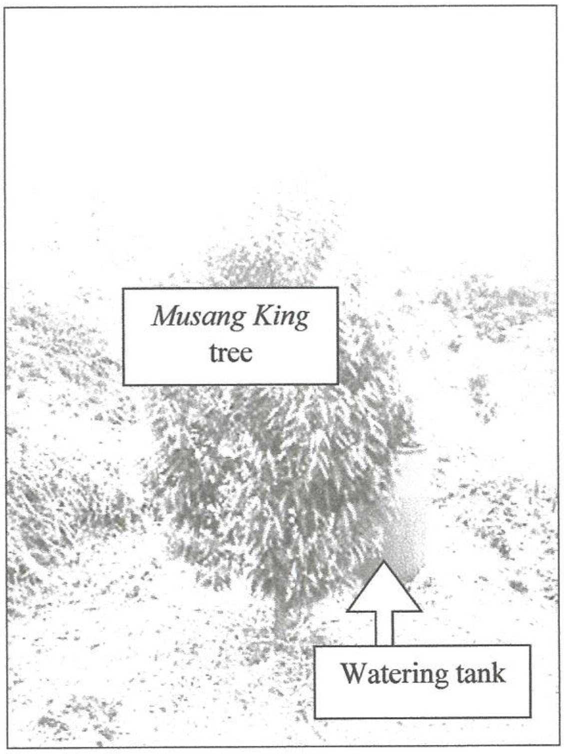


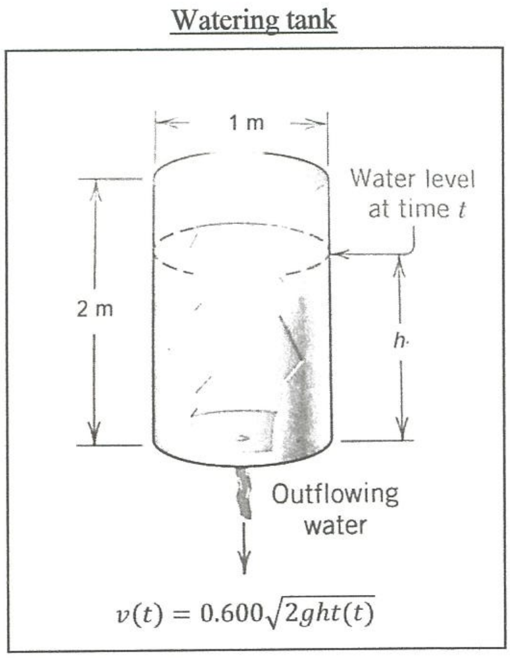


**Figure Q1**

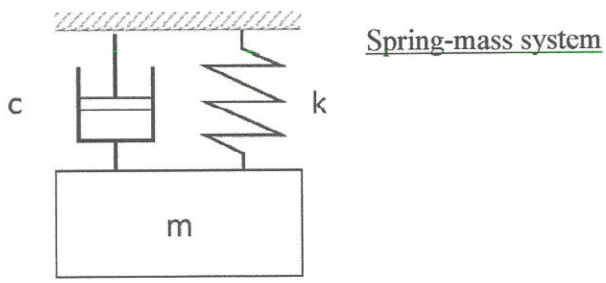


**Figure Q2**

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## FORMULAS

**First Order Differential Equations**

| Type of ODEs | General solution |
|---|---|
| Linear ODEs: $y' + P(x)y = Q(x)$ | $y = e^{-\int P(x)dx}\left\{\int e^{\int P(x)dx}Q(x)dx + C\right\}$ |
| Exact ODEs: $f(x,y)dx + g(x,y)dy = 0$ | $F(x,y) = \int f(x,y)dx$<br>$F(x,y) - \int \left\{\frac{\partial F}{\partial y} - g(x,y)\right\} dy = C$ |
| Inexact ODEs: $M(x,y)dx + N(x,y)dy = 0$<br>$\frac{\partial M}{\partial y} \neq \frac{\partial N}{\partial x}$<br>Integrating factor;<br>$i(x) = e^{\int f(x)dx}$ where $f(x) = \frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right)$<br>$i(y) = e^{\int g(y)dy}$ where $g(y) = \frac{1}{M}\left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)$ | $\int iM(x,y)dx - \int \left\{\frac{\partial\left(\int iM(x,y)dx\right)}{\partial y} - iN(x,y)\right\}dy = C$ |

**Characteristic Equation and General Solution for Second Order Differential Equations**

| Types of Roots | General Solution |
|---|---|
| Real and Distinct Roots: $m_1$ and $m_2$ | $y = c_1 e^{m_1 x} + c_2 e^{m_2 x}$ |
| Real and Repeated Roots: $m_1 = m_2 = m$ | $y = c_1 e^{mx} + c_2 x e^{mx}$ |
| Complex Conjugate Roots: $m = \alpha \pm i\beta$ | $y = e^{\alpha x}(c_1 \cos \beta x + c_2 \sin \beta x)$ |

**Method of Undetermined Coefficients**

| $g(x)$ | $y_p$ |
|---|---|
| **Polynomial:** $P_n(x) = a_n x^n + \ldots + a_1 x + a_0$ | $x^r(A_n x^n + \ldots + A_1 x + A_0)$ |
| **Exponential:** $e^{ax}$ | $x^r(Ae^{ax})$ |
| **Sine or Cosine:** $\cos \beta x$ or $\sin \beta x$ | $x^r(A\cos \beta x + B\sin \beta x)$ |

*Note*: $r$ is 0, 1, 2 … in such a way that no terms in $y_p(x)$ is similar to those in $y_c(x)$.

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**Method of Variation of Parameters**

The particular solution for $y'' + by' + cy = g(x)$ with $b$ and $c$ as constants is:

$y(x) = u_1 y_1 + u_2 y_2$

where,

$u_1 - -\int \frac{u_1 g(x)}{W} dx$ & $u_2 = \int \frac{u_2 g(x)}{W} dx$

where,

$W = \begin{vmatrix} y_1 & y_2 \\ y'_1 & y'_2 \end{vmatrix}$

**Laplace Transform**

$$\mathcal{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

| $f(t)$ | $F(s)$ |
|---|---|
| $a$ | $\frac{a}{s}$ |
| $t^n, n = 1,2,3,\ldots$ | $\frac{n!}{s^{n+1}}$ |
| $e^{at}$ | $\frac{1}{s-a}$ |
| $\sin at$ | $\frac{a}{s^2+a^2}$ |
| $\cos at$ | $\frac{s}{s^2+a^2}$ |
| $\sinh at$ | $\frac{a}{s^2-a^2}$ |
| $\cosh at$ | $\frac{s}{s^2-a^2}$ |
| $e^{at} f(t)$ | $F(s-a)$ |
| $t^n f(t), n = 1,2,3,\ldots$ | $(-1)^n \frac{d^n F(s)}{ds^n}$ |

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**Laplace Transform (continued)**

| | |
|---|---|
| $H(t-a)$ | $\dfrac{e^{-as}}{s}$ |
| $f(t-a)H(t-a)$ | $e^{-as}F(s)$ |
| $f(t)\delta(t-a)$ | $e^{-as}f(a)$ |
| $y(t)$ | $Y(s)$ |
| $\dot{y}(t)$ | $sY(s)-y(0)$ |
| $\ddot{y}(t)$ | $s^2Y(s)-sy(0)-y'(0)$ |

**Partial Fraction Expansion**

| Factor in denominator | Term in partial fraction decomposition |
|---|---|
| $ax+b$ | $\dfrac{A}{ax+b}$ |
| $(ax+b)^k$ | $\dfrac{A_1}{ax+b}+\dfrac{A_2}{(ax+b)^2}+\cdots+\dfrac{A_k}{(ax+b)^k},\ k=1,2,3,\ldots$ |
| $ax^2+bx+c$ | $\dfrac{Ax+B}{ax^2+bx+c}$ |
| $(ax^2+bx+c)^k$ | $\dfrac{A_1x+B_1}{ax^2+bx+c}+\dfrac{A_2x+B_2}{(ax^2+bx+c)^2}+\cdots+\dfrac{A_kx+B_k}{(ax^2+bx+c)^k},\ k=1,2,3,\ldots$ |

**Fourier Series**

Fourier series expansion of periodic function with period $2\pi$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx$$

$$b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx$$

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos nx+\sum_{n=1}^{\infty}b_n\sin nx$$

Half Range Series

$$a_0=\frac{2}{L}\int_0^L f(x)dx$$

$$a_n=\frac{2}{L}\int_0^L f(x)\cos\frac{n\pi x}{L}dx$$

$$b_n=\frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}dx$$

$$f(x)=\frac{1}{2}a_0+\sum_{n=1}^{\infty}a_n\cos\frac{n\pi x}{L}+\sum_{n=1}^{\infty}b_n\sin\frac{n\pi x}{L}$$

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## Trigonometric Identities

### TANGENT IDENTITIES

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

### RECIPROCAL IDENTITIES

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sin\theta = \frac{1}{\csc\theta}$$

$$\sec\theta = \frac{1}{\cos\theta} \qquad \cos\theta = \frac{1}{\sec\theta}$$

$$\cot\theta = \frac{1}{\tan\theta} \qquad \tan\theta = \frac{1}{\cot\theta}$$

### PYTHAGOREAN IDENTITIES

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$\cot^2\theta + 1 = \csc^2\theta$$

### PERIODIC IDENTITIES

$$\sin(\theta + 2\pi n) = \sin\theta$$

$$\cos(\theta + 2\pi n) = \cos\theta$$

$$\tan(\theta + \pi n) = \tan\theta$$

$$\csc(\theta + 2\pi n) = \csc\theta$$

$$\sec(\theta + 2\pi n) = \sec\theta$$

$$\cot(\theta + \pi n) = \cot\theta$$

### EVEN/ODD IDENTITIES

$$\sin(-\theta) = -\sin\theta$$

$$\cos(-\theta) = \cos\theta$$

$$\tan(-\theta) = -\tan\theta$$

$$\csc(-\theta) = -\csc\theta$$

$$\sec(-\theta) = \sec\theta$$

$$\cot(-\theta) = -\cot\theta$$

### DOUBLE ANGLE IDENTITIES

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

$$\tan(2\theta) = \frac{2\tan\theta}{1 - \tan^2\theta}$$

### HALF ANGLE IDENTITIES

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}$$

### LAW OF COSINES

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$b^2 = a^2 + c^2 - 2ac\cos\beta$$

$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

### PRODUCT TO SUM IDENTITIES

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

$$\cos\alpha\sin\beta = \frac{1}{2}[\sin(\alpha+\beta) - \sin(\alpha-\beta)]$$

### SUM TO PRODUCT IDENTITIES

$$\sin\alpha + \sin\beta = 2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\sin\alpha - \sin\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

### LAW OF TANGENTS

$$\frac{a-b}{a+b} = \frac{\tan\left[\frac{1}{2}(\alpha-\beta)\right]}{\tan\left[\frac{1}{2}(\alpha+\beta)\right]}$$

$$\frac{b-c}{b+c} = \frac{\tan\left[\frac{1}{2}(\beta-\gamma)\right]}{\tan\left[\frac{1}{2}(\beta+\gamma)\right]}$$

$$\frac{a-c}{a+c} = \frac{\tan\left[\frac{1}{2}(\alpha-\gamma)\right]}{\tan\left[\frac{1}{2}(\alpha+\gamma)\right]}$$

### SUM/DIFFERENCES IDENTITIES

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

### MOLLWEIDE'S FORMULA

$$\frac{a+b}{c} = \frac{\cos\left[\frac{1}{2}(\alpha-\beta)\right]}{\sin\left(\frac{1}{2}\gamma\right)}$$

### COFUNCTION IDENTITIES

$$\sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$$

$$\csc\left(\frac{\pi}{2} - \theta\right) = \sec\theta$$

$$\tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$$

$$\cos\left(\frac{\pi}{2} - \theta\right) = \sin\theta$$

$$\sec\left(\frac{\pi}{2} - \theta\right) = \csc\theta$$

$$\cot\left(\frac{\pi}{2} - \theta\right) = \tan\theta$$

### LAW OF SINES

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c}$$