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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER II
SESSION 2021/2022

COURSE NAME	:	ENGINEERING MATHEMATICS
COURSE CODE	:	BFC 25103
PROGRAMME	:	BFF
EXAMINATION DATE	:	JULY 2022
DURATION	:	3 HOURS
INSTRUCTION	1.	ANSWER ALL QUESTIONS
	2.	THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **CLOSE BOOK**.
	3.	STUDENTS ARE **PROHIBITED** TO CONSULT THEIR OWN MATERIAL OR ANY EXTERNAL RESOURCES DURING THE EXAMINATION CONDUCTED VIA CLOSED BOOK

THIS QUESTION PAPER CONSISTS OF **ELEVEN (11)** PAGES

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Q1 (a) Determine the Inverse Laplace transform using partial functions for the following:

i) $\mathcal{L}^{-1}\{\frac{4s^2-5s+6}{(s+1)(s^2+4)}\}$

(5 marks)

ii) $\mathcal{L}^{-1}\{\frac{4s^2-17s-24}{(s^3-s^2-12s)}\}$

(5 marks)

(b) A function g is defined by

$$g(x) = \begin{cases} -11 + ax & x \leq -2 \\ b + cx^3 & -2 < x \leq 1 \\ 4 - x^2 & x > 1 \end{cases}$$

i) Calculate the value of a if $\lim_{x \to -2} g(x) = -15$

(2 marks)

ii) Estimate the value of b and c if $\lim_{x \to -2} g(x)$ and $\lim_{x \to 1} g(x)$ exist.

(3 marks)

(c) Solve the partial derivatives below:

i) According to the ideal gas law, the pressure, temperature, $T(K)$ and volume, $V(m^3)$ of a gas are related by

$$P = \frac{kT}{V}$$

where k is a constant of proportionality, $k = 10\, m \cdot lb/K$. Determine the instantaneous rate of change of pressure with respect to temperature and the instantaneous rate of change of volume with respect to pressure, if the temperature is 80K and the volume remains fixed at $40m^3$.

(5 marks)

ii) Suppose that $D = \sqrt{x^2 + y^2}$ is the length of rectangle which has lengths x and y. Find a formula for the rate of change of D with respect to x if x varies with y hold constant and use this formula to find the rate of change of D with respect to x at the point where $x = 3$ and $y = 4$.

(5 marks)

Q2 (a) A radar, with coordinates (0,0), has coverage with length of ***d***. While the second radar, with identical coverage, is situated on the east side of the first one. Using signal intersection, both radars detect an object coming closer to the southern direction in the first quadrant. Note that radars swap the covered area into circles.

i) Identify position of the object in terms of distance and angle. Complete your answer with a sketch.

(5 marks)

ii) Analyze and calculate the overlapping area from the radar signals intersection at the first quadrant.

(10 marks)

(b) A cylinder C is bounded by $x^2 + y^2 \leq 9$ and $-1 \leq z \leq 2$. The cylinder is evaluated by using triple integral in a form of:

$$\iiint_C z\sqrt{x^2 + y^2}\, dx\, dy\, dz$$

i) Express the equation $z\sqrt{x^2 + y^2}$ in term of cylindrical coordinates as well as its respective integral upper-lower bounds, C.

(5 marks)

ii) Solve the triple integral in cylindrical coordinates system.

(5 marks)

Q3 (a) An engineer is designing a right-angle triangular roof as shown in **FIGURE Q3 (a)** with one column. In order to make sure that the roof is stable and safe, the centre of mass need to be calculated before the design stage. Determine mass, moment of mass and centre of mass of the roof. Assume that the density is x.

(8 marks)

(b) Estimate the vector value function

$$\mathbf{r}(t) = 5\cos(t)\mathbf{i} + 5sin(t)\mathbf{j} + t\mathbf{k}$$

if $t = 0$ and $t = \pi/2$. Then, sketch the graph of the vector value function.

(5 marks)

(c) Calculate the domain and limit $(t \to 0)$ of the vector-valued function $\mathbf{r}(t) = \sqrt[4]{t}\mathbf{i} + sin\, 5t\, \mathbf{j} + ln\, 5t$.

(4 marks)

(d) For the given function, $\mathbf{r}(t) = \frac{1}{t}\mathbf{i} + t\cos 3t\, \mathbf{j} + 6\mathbf{k}$

i) Calculate $r'(t)$ and $\int r(t)$

(6 marks)

ii) After integration, three constants are introduced for each vector component. Based on your opinion, are these three constants similar or different. Explain your answer.

(2 marks)

Q4 (a) Calculate the unit tangent vector, principal unit normal vector, binormal vector and curvature of vector valued functions; $r(t) = 2\cos\frac{\pi}{2}t\,i + 2\sin\frac{\pi}{2}t\,j - 2\,k$

(10marks)

(b) Given that the line integral equation of $\int_C xy\,dx + (x+y)\,dy$ where C is the curve, calculate;

i) A straight line from the point (0,0) to (1,1)

(3 marks)

ii) $x = \sqrt{y}$ from the point (0,0) to (1,1)

(3 marks)

(c) Compute the flux of water through the cone $z = 1 - \sqrt{x^2 + y^2}$ that line above the plane $z = 0$, oriented by an upward unit normal vector. Assume that the velocity vector, $v = F(x, y, z) = 2\,x\,i + 2\,y\,j + 3\,k$ is measured in m/min and the water has the density $\rho = 1\,tan/m^3$.

(9 marks)

-END OF QUESTIONS-

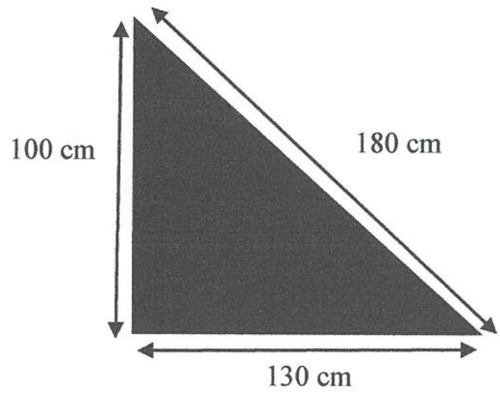

Figure Q3 (a): Right-angle triangular roof

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

Laplace Transforms

$$\mathbf{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}\,dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}

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$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathbf{L}\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt,\quad s>0.$

Table .1 Inverse Laplace transforms

$F(s)=\mathcal{L}[f(t)]$		$\mathcal{L}^{-1}[F(s)]=f(t)$
(i)	$\frac{1}{s}$	1
(ii)	$\frac{k}{s}$	k
(iii)	$\frac{1}{s-a}$	e^{at}
(iv)	$\frac{a}{s^2+a^2}$	$\sin at$
(v)	$\frac{s}{s^2+a^2}$	$\cos at$
(vi)	$\frac{1}{s^2}$	t
(vii)	$\frac{2!}{s^3}$	t^2
(viii)	$\frac{n!}{s^{n+1}}$	t^n
(ix)	$\frac{a}{s^2-a^2}$	$\sinh at$
(x)	$\frac{s}{s^2-a^2}$	$\cosh at$
(xi)	$\frac{n!}{(s-a)^{n+1}}$	$e^{at}t^n$
(xii)	$\frac{\omega}{(s-a)^2+\omega^2}$	$e^{at}\sin \omega t$
(xiii)	$\frac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos \omega t$
(xiv)	$\frac{\omega}{(s-a)^2-\omega^2}$	$e^{at}\sinh \omega t$
(xv)	$\frac{s-a}{(s-a)^2-\omega^2}$	$e^{at}\cosh \omega t$

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Formulae

Tangent Plane: $z - z_0 = f_x(x_o, y_0)(x - x_0) + f_y(x_o, y_0)(y - y_0)$

Local Extreme Value: $G(x, y) = f_{xx}(x, y) \times f_{yy}(x, y) - [f_{xy}(x, y)]^2$

Case	$G(a,b)$	Result
1	$G(a,b) > 0$ $f_{xx}(a,b) < 0$	$f(x,y)$ has a local maximum value at (a,b)
2	$G(a,b) > 0$ $f_{xx}(a,b) > 0$	$f(x,y)$ has a local minimum value at (a,b)
3	$G(a,b) < 0$	$f(x,y) has\ a\ saddle\ point\ at\ (a,b)$
4	$G(a,b) = 0$	inconclusive

Polar coordinate: $x = r\cos\theta, y = r\sin\theta, \theta = \tan^{-1}(\frac{y}{x})$ and
$\iint_R f(x,y)dA = \iint_R f(r,\theta)r dr d\theta$

Cylindrical coordinate: $x = r\cos\theta,$, $y = r\sin\theta,$ $z = z,$
$\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)d\ dz\ dr\ d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta, y = \rho\sin\phi\sin\theta, z = \rho\cos\theta, x^2 + y^2 + z^2 = \rho^2,$
$0 \ll \theta \ll 2\pi, 0 \ll \phi \ll \pi$ and $\iiint f(x,y,z)dV = \iiint f(\rho,\phi,\theta)\rho^2 \sin\phi\ d\rho d\phi d\theta$

For lamina

Mass, $m = \iint_R \delta(x,y)\ dA$

Moment of mass: y-axis: $M_y = \iint_R x\delta(x,y)\ dA$ x-axis, $M_x = \iint_R y\delta(x,y)\ dA$

Center of mass, $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$

Centroid for homogenous lamina: $\bar{x} = \frac{1}{area}\iint_R x\ dA$ $\bar{y} = \frac{1}{area}\iint_R y\ dA$

Moment inertia:

Y-axis: $I_y = \iint_R x^2\,\delta(x,y)\ dA$ x-axis: $I_x = \iint_R y^2\,\delta(x,y)\ dA$

Z-axis (or origin): $I_z = I_0 = \iint_R (x^2 + y^2)\,\delta(x,y)\ dA = I_x + I_y$

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For solid

Mass, $m = \iiint_G \delta(x,y)\, dV$

Moment of mass:

yz-plane: $M_{yz} = \iiint_G x\, \delta(x,y,z)\, dV$

xz-plane: $M_{xz} = \iiint_G y\, \delta(x,y,z)\, dV$

xy-plane: $M_{xy} = \iiint_G z\, \delta(x,y,z)\, dV$

Center of gravity, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m})$

Moment inertia:

$$I_y = \iiint_G (x^2 + z^2)\, \delta(x,y,z)\, dV$$

$$I_x = \iiint_G (y^2 + z^2)\, \delta(x,y,z)\, dV$$

$$I_z = \iiint_G (x^2 + y^2)\, \delta(x,y,z)\, dV$$

Directional derivative: $D_u f(x,y) = (f_x\mathbf{i} + f_y\mathbf{j}) \cdot u$

Let $\mathbf{F}(x,y,z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then the divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The curl of

$$\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let C is a smooth curve given by $r(t) = x(t)i + y(t)j + z(t)k, t$ is parameter, then

The unit tangent vector; $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The binormal vector: $B(t) = T(t) \times N(t)$

The curvature: $K = \frac{\mathbf{T}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}' \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

The radius of curvature: $\rho = 1/K$

Green Theorem: $\oint_C M(x,y)\, dx + N(x,y)\, dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y} dA$

Gauss Theorem: $\iint_\sigma \mathbf{F} \cdot \mathbf{n}\, dS = \iiint_G \nabla . \mathbf{F}\, dV$

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Stokes Theorem: $\oint_C \mathbf{F}.d\mathbf{r} = \iint_\sigma (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$

Arc length, If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\, dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\, dt$$

If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\, dt$$

Trigonometric and Hyperbolic Identities

Trigonometric	**Hyperbolic**
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \mathrm{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \mathrm{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \mathrm{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$
$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1 + \tanh^2 x}$

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$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	