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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	CIVIL ENGINEERING MATHEMATIC III
COURSE CODE	:	BFC 24103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS. 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **CLOSE BOOK.**

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 (a) Sketch the domain of the $f(x, y) = \frac{x-2y}{x+2y}$, and find the range of the function.

(5 marks)

(b) Given $z = x^2 + y^2,\ where\ x = \frac{1}{t^2}, and\ y = 2t^3$. Find $\frac{dz}{dt}$ using chain rule.

(10 marks)

(c) Find the partial derivatives of $f(x, y, z) = x^2\, yz^{-3}, and\ f(x, y, z) = \ln\left(\frac{x}{y}\right) + \tan^{-1}(yz)$.

(10 marks)

Q2 (a) By changing to polar coordinate, evaluate $\iint_R x^2 + y^2\, dA$, if R is the region in between circle $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$ in the first octant.

(10 marks)

(b) Determine the volume of solid bounded, above by $z = x^2 + y^2$, below by xy-plane and side by cylinder $x^2 + y^2 = 9$ and in the first octant.

(5 marks)

(c) By using spherical coordinate, evaluate and find the – volume of the solid bounded above by sphere $x^2 + y^2 + z^2 = 4$ + and below by cone $z = \sqrt{x^2 + y^2}$.

(10 marks)

Q3 (a) Evaluating the limit of function component by component. Determine limit of vector-valued function of $\lim_{t \to 0}(t^2\mathbf{i} + \sqrt{4 - t}\,\mathbf{j} + \ln t\,\mathbf{k})$.

(5 marks)

(b) Calculate its unit tangent vector and principal unit normal vector at $t = {}^{\pi}/_{4}$.. Then, sketch the graph of $\mathbf{r}(t)$, $\mathbf{T}(\pi)$ and $\mathbf{N}(\pi)$ in the same axis, when the vector-value function is given as $\mathbf{r}(t) = 4\cos t\,\mathbf{i} + 4\sin t\ \mathbf{j}$.

(10 marks)

(c) Find the velocity, speed and acceleration of the particle at $t = \pi$ with the position vector $r(t) = tcost\,\mathbf{i} + e^t sint\,\mathbf{j} + 2t\,\mathbf{k}$.

(10 marks)

Q4 (a) Let σ be the surface of the plane $z = xy$ with $0 \leq x \leq 1, 0 \leq y \leq 2$ and $\mathbf{F}(x, y, z) = xz\,\mathbf{j} + xy\,\mathbf{k}$ across σ. The $\mathbf{n}$ is the unit normal vector oriented outward. Evaluate $\iint_R \mathbf{F}.\mathbf{n}\, ds$.

(9 marks)

(b) Find the directional derivative of the function $f(x, y, z) = e^{2x+y+3z}$ at the point $(0, 1, -1)$ in the direction of vector $\vec{a} = 2\mathbf{i} + 2\mathbf{j} + \mathbf{k}$.

(8 marks)

(c) Determine a unit vector in the direction in which $f(x, y) = 4x^3\, y^2$ increases most rapidly at point P(-1,1).

(8 marks)

- END OF QUESTIONS –

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Formulae

Chain rule.

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt}$$

First partial derivatives

$$f_x(x,y) = \lim_{h\to 0}\frac{f(x+h,y) - f(x,y)}{h}$$

$$f_y(x,y) = \lim_{h\to 0}\frac{f(x,h+y) - f(x,y)}{h}$$

Cylinder coordinate

$$Volume = \iiint dv = \iiint dz\, r\, dr\, d\theta$$

Spherical coordinate

$$Volume = \iiint dv = \iiint \rho^2 \sin\emptyset\, d\rho\, d\emptyset\, d\theta$$

Limit of vector-valued function

$$\lim_{t\to a} r(t) = \left(\lim_{t\to a} f(t)\right)\mathbf{i} + \left(\lim_{t\to a} g(t)\right)\mathbf{j} + \left(\lim_{t\to a} h(t)\right)\mathbf{k}$$

Unit tangent vector

$$\mathbf{T}(t) = \frac{r'(t)}{\|r'(t)\|}$$

Principal unit normal vector

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$$

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Velocity

$$V(t) = r'(t)$$

$$V(t) = \frac{d\boldsymbol{r}(t)}{dt}$$

Speed

$$= \|V(t)\|$$

Acceleration

$$a(t) = v'(t) = r''(t)$$

$$a(t) = \frac{dv(t)}{dt} + \frac{d^2r(t)}{dt^2}$$

Surface integral of vector fields

$$\iint_R \mathbf{F}.\mathbf{n}\, ds.$$

Oriented outward

$$\boldsymbol{n} = \frac{\nabla\emptyset}{|\nabla\emptyset|} = \frac{-\frac{\partial z}{\partial x}\,\boldsymbol{i} - \frac{\partial z}{\partial y}\,\mathbf{j} + \boldsymbol{k}}{\sqrt{\left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2 + 1}}$$

Directional derivative

$$D_u f(x, y, z) = (f_x\, i + f_y\, j + f_z k).(u_a\, i + u_2\, j + u_3\, k)$$

Unit vector in the direction

$$\nabla f = \frac{\partial f}{\partial x} i + \frac{\partial f}{\partial y} j$$