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UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	ENGINEERING MATHEMATICS
COURSE CODE	:	BFC 25103
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **ALL** QUESTIONS. 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **CLOSE BOOK**.

THIS QUESTION PAPER CONSISTS OF **TEN (10)** PAGES

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Q1 (a) Given that $L\{e^{at}\} = \frac{1}{s-a}$ by using the formula $L\{t^n f(t)\} = (-1)^n \frac{d^n}{s^n}\ [F(s)]$ find the following functions;

(i) $L\{te^{at}\}$

(2 marks)

(ii) $L\{t^2 e^{at}\}$

(2 marks)

(b) By applying the first shift theorem, calculate the following functions;

(i) $L = \{e^{-2t} cos\ 6t\}$

(3 marks)

(ii) Based on your understanding, explain why the first shift theorem needs to be applied to solve the function.

(2 marks)

(c) Determine the Inverse Laplace transforms using partial fractions for the following equations;

(i) $L^{-1}\{\frac{3s^3+s^2+12s+2}{(s-3)(s+1)^3}\}$

(3 marks)

(ii) $L^{-1}\{\frac{7s+13}{s(s^2+4s+13)}\}$

(3 marks)

(d) A school with two floors (ground and first floor), the first floor has mass m = 1459.39 kg and weighs =14515 kg. The elastic frame of the building has a spring to resist the horizontal displacements of the first floor;where 6.25 tons of a horizontal force is required to displace 50 cm of the first floor. In one of the earthquake, the ground oscillated horizontally with amplitude A_0 and circular frequency ω, leading to an external horizontal force $F(t) = mA_0w^2 \sin(wt)$ on the first floor.

(i) Apply the second (2^{nd}) order equations to compute the natural frequency (in hertz) of oscillations of the first floor

(5 marks)

(ii) Calculate the amplitude of the resulting forced oscillations of the first floor, if the ground undergoes one oscillation every 1.37s with an amplitude of 3.8 cm.

(5 marks)

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Q2 (a) Determine the partial differentiation for each of the following functions

(i) $z = x^2y\,cos(xy)$

(4 marks)

(ii) $z = (3x^2 + y)^2$

(4 marks)

(b) Classify the given function as continuous or not continuous

$$f(x,y) = \begin{cases} \dfrac{x^2 - y^2}{x^2 + 2y}, & if\ (x,y) \neq (0,0) \\ 0 & if\ (x,y) = (0,0) \end{cases}$$

(5 marks)

(c) Radius of a right circular cylinder is measured with an error of at most 5%, and the height is measured with an error of at most 9%.

(i) Solve the maximum possible percentage error in the volume calculated from these measurements.

(9 marks)

(ii) If the height is measured with an error at most 8%. Compare the maximum possible percentage error with an error at most 9%. Justify your answer.

(3 marks)

Q3 (a) By using double integrals equations, find

$$\iint_R e^{x2}\,\mathrm{dA}, \text{ where R is the region between x} - \text{axis, lines } y = \frac{x}{2} \text{ and x} = 4$$

(6 marks)

(b) Determine the centre of mass of the triangle with boundaries $y = 0$, $x = 1$ and $y = 2x$, and mass density $\rho(x,y) = x + y$.

(5 marks)

(c) By applying polar coordinates, calculate;

$$\iint_R x + y\,dxdy \text{ over a region bounded by curves } xy = 6\ and\ x + y = 7 \text{ and Sketch a diagram of the region.}$$

(8 marks)

(d) Solve the triple integral of $f\ (x,y,z) = z$ in the first octant and bounded by $0 \leq x, 3x \leq y, 0 \leq z\ and\ y^2 + z^2 \leq 9$.

(6 marks)

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Q4 (a) If $r(t) = 3\cos t\, i + 3\sin t\, j + 3t\, k$, find the length of the arc of the circular helix from the point (1, 0, 0) to the point (1, 0, 6).

(6 marks)

(b) Based on your opinion, explain why the curvature of a circle of radius a is $1/a$.

(6 marks)

(c) Calculate the unit normal and binomial vectors for the circular helix; $r(t) = 3\cos t\, i + 3\sin t\, j + 3t\, k$

(6 marks)

(d) Solve $\int C\, \vec{F}.d\vec{r}$ where $\vec{F}(x, y, z) = 8x^2yz\vec{i} + 5z\vec{j} - 4xy\vec{k}$ and C is the curve given by $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}, 0 \leq t \leq 1$.

(7 marks)

-END OF QUESTIONS-

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Formulae

Characteristic Equation and General Solution

Case	Roots of the Characteristic Equation	General Solution
1	m_1 and m_2 ; real and distinct	$y = Ae^{m_1 x} + Be^{m_2 x}$
2	$m_1 = m_2 = m$; real and equal	$y = (A + Bx)e^{mx}$
3	$m = \alpha \pm i\beta$; imaginary	$y = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Undetermined Coefficients

$f(x)$	$y_p(x)$
$P_n(x) = A_n x^n + \cdots + A_1 x + A_0$	$x^r(B_n x^n + \cdots + B_1 x + B_0)$
$Ce^{\alpha x}$	$x^r(Pe^{\alpha x})$
$C\cos\beta x$ or $C\sin\beta x$	$x^r(p\cos\beta x + q\sin\beta x)$

Particular Integral of $ay'' + by' + cy = f(x)$: Method of Variation of Parameters

Wronskian	Parameter	Solution
$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$	$u_1 = -\int \frac{y_2 f(x)}{W} dx, \quad u_2 = \int \frac{y_1 f(x)}{W} dx$	$y_p = u_1 y_1 + u_2 y_2$

Laplace Transforms

$$\mathbf{L}\{f(t)\} = \int_0^\infty f(t)e^{-st}dt = F(s)$$

$f(t)$	$F(s)$	$f(t)$	$F(s)$
a	$\frac{a}{s}$	$H(t-a)$	$\frac{e^{-as}}{s}$
t^n, $n = 1, 2, 3, \ldots$	$\frac{n!}{s^{n+1}}$	$f(t-a)H(t-a)$	$e^{-as}F(s)$
e^{at}	$\frac{1}{s-a}$	$\delta(t-a)$	e^{-as}

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$\sin at$	$\frac{a}{s^2+a^2}$	$f(t)\delta(t-a)$	$e^{-as}f(a)$
$\cos at$	$\frac{s}{s^2+a^2}$	$\int_0^t f(u)g(t-u)du$	$F(s).G(s)$
$\sinh at$	$\frac{a}{s^2-a^2}$	$y(t)$	$Y(s)$
$\cosh at$	$\frac{s}{s^2-a^2}$	$\dot{y}(t)$	$sY(s)-y(0)$
$e^{at}f(t)$	$F(s-a)$	$\ddot{y}(t)$	$s^2Y(s)-sy(0)-\dot{y}(0)$
$t^n f(t),\ n=1,2,3,\ldots$	$(-1)^n \frac{d^nF(s)}{ds^n}$		

Periodic Function for Laplace transform : $\mathrm{L}\{f(t)\}=\frac{1}{1-e^{-sT}}\int_0^T e^{-st}f(t)\,dt,\quad s>0.$

Table .1 Inverse Laplace transforms

	$F(s)=\mathcal{L}[f(t)]$	$\mathcal{L}^{-1}[F(s)]=f(t)$
(i)	$\frac{1}{s}$	1
(ii)	$\frac{k}{s}$	k
(iii)	$\frac{1}{s-a}$	e^{at}
(iv)	$\frac{a}{s^2+a^2}$	$\sin at$
(v)	$\frac{s}{s^2+a^2}$	$\cos at$
(vi)	$\frac{1}{s^2}$	t
(vii)	$\frac{2!}{s^3}$	t^2
(viii)	$\frac{n!}{s^{n+1}}$	t^n
(ix)	$\frac{a}{s^2-a^2}$	$\sinh at$
(x)	$\frac{s}{s^2-a^2}$	$\cosh at$
(xi)	$\frac{n!}{(s-a)^{n+1}}$	$e^{at}t^n$
(xii)	$\frac{\omega}{(s-a)^2+\omega^2}$	$e^{at}\sin \omega t$
(xiii)	$\frac{s-a}{(s-a)^2+\omega^2}$	$e^{at}\cos \omega t$
(xiv)	$\frac{\omega}{(s-a)^2-\omega^2}$	$e^{at}\sinh \omega t$
(xv)	$\frac{s-a}{(s-a)^2-\omega^2}$	$e^{at}\cosh \omega t$

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Formulae

Tangent Plane: $z - z_0 = f_x(x_o, y_0)(x - x_0) + f_y(x_o, y_0)(y - y_0)$

Local Extreme Value: $G(x,y) = f_{xx}(x,y) \times f_{yy}(x,y) - [f_{xy}(x,y)]^2$

Case	$G(a,b)$	Result
1	$G(a,b) > 0$ $f_{xx}(a,b) < 0$	$f(x,y)$ has a local maximum value at (a,b)
2	$G(a,b) > 0$ $f_{xx}(a,b) > 0$	$f(x,y)$ has a local minimum value at (a,b)
3	$G(a,b) < 0$	$f(x,y) has\ a\ saddle\ point\ at\ (a,b)$
4	$G(a,b) = 0$	inconclusive

Polar coordinate: $x = r\cos\theta$, $y = r\sin\theta$, $\theta = tan^{-1}(\frac{y}{x})$ and
$\iint_R f(x,y)dA = \iint_R f(r,\theta)rdrd\theta$

Cylindrical coordinate: $x = r\cos\theta$, , $y = r\sin\theta$, $z = z$, $\iiint_G f(x,y,z)dV = \iiint_G f(r,\theta,z)d\ dz\ dr\ d\theta$

Spherical coordinate: $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$, $z = \rho\cos\theta$, $x^2 + y^2 + z^2 = \rho^2$, $0 \ll \theta \ll 2\pi$, $0 \ll \phi \ll \pi$ and $\iiint f(x,y,z)dV = \iiint f(\rho,\phi,\theta)\rho^2\sin\phi\ d\rho d\phi d\theta$

For lamina

Mass, $m = \iint_R \delta(x,y)\ dA$

Moment of mass: y-axis: $M_y = \iint_R x\delta(x,y)\ dA$ x-axis, $M_x = \iint_R y\delta(x,y)\ dA$

Center of mass, $(\bar{x}, \bar{y}) = (\frac{M_y}{m}, \frac{M_x}{m})$

Centroid for homogenous lamina: $\bar{x} = \frac{1}{area}\iint_R x\ dA$ $\bar{y} = \frac{1}{area}\iint_R y\ dA$

Moment inertia:

Y-axis: $I_y = \iint_R x^2\,\delta(x,y)\ dA$ x-axis: $I_x = \iint_R y^2\,\delta(x,y)\ dA$

Z-axis (or origin): $I_z = I_0 = \iint_R (x^2 + y^2)\,\delta(x,y)\ dA = I_x + I_y$

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For solid

Mass, $m = \iiint_G \delta(x, y)\, dV$

Moment of mass:

yz-plane: $M_{yz} = \iiint_G x\,\delta(x, y, z)\, dV$

xz-plane: $M_{xz} = \iiint_G y\,\delta(x, y, z)\, dV$

xy-plane: $M_{xy} = \iiint_G z\,\delta(x, y, z)\, dV$

Center of gravity, $(\bar{x}, \bar{y}, \bar{z}) = (\frac{M_{yz}}{m}, \frac{M_{xz}}{m}, \frac{M_{xy}}{m})$

Moment inertia:

$$I_y = \iiint_G (x^2 + z^2)\, \delta(x, y, z)\, dV$$

$$I_x = \iiint_G (y^2 + z^2)\, \delta(x, y, z)\, dV$$

$$I_z = \iiint_G (x^2 + y^2)\, \delta(x, y, z)\, dV$$

Directional derivative: $D_u f(x, y) = (f_x \mathbf{i} + f_y \mathbf{j}) \cdot u$

Let $\mathbf{F}(x, y, z) = M\mathbf{i} + N\mathbf{j} + P\mathbf{k}$ is vector field, then the divergence of $\mathbf{F} = \nabla \cdot \mathbf{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z}$

The curl of

$$\mathbf{F} = \nabla \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ M & N & P \end{vmatrix} = \left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\mathbf{i} - \left(\frac{\partial P}{\partial x} - \frac{\partial M}{\partial z}\right)\mathbf{j} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\mathbf{k}$$

Let C is a smooth curve given by $r(t) = x(t)i + y(t)j + z(t)k$, t is parameter, then

The unit tangent vector; $\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{\|\mathbf{r}'(t)\|}$

The unit normal vector: $\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{\|\mathbf{T}'(t)\|}$

The binormal vector: $B(t) = T(t) \times N(t)$

The curvature: $K = \frac{\mathbf{T}'(t)}{\|\mathbf{r}'(t)\|} = \frac{\|\mathbf{r}' \times \mathbf{r}''(t)\|}{\|\mathbf{r}'(t)\|^3}$

The radius of curvature: $\rho = 1/K$

Green Theorem: $\oint_C M(x, y)\, dx + N(x, y)\, dy = \iint_R \frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\, dA$

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Gauss Theorem: $\iint_{\sigma} \mathbf{F}\cdot\mathbf{n}\; dS = \iiint_{G} \nabla.\mathbf{F}\; dV$

Stokes Theorem: $\oint_{C} \mathbf{F}.d\mathbf{r} = \iint_{\sigma} (\nabla\times\mathbf{F})\cdot\mathbf{n}\; dS$

Arc length, If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \|\mathbf{r}'(t)\|\, dt = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2}\; dt$$

If $r(t) = x(t)\mathbf{i} + y(t)\mathbf{j} + z(t)\mathbf{k}$, $t\epsilon[a,b]$, then the arc length

$$s = \int_a^b \sqrt{(x'(t))^2 + (y'(t))^2 + (z'(t))^2}\; dt$$

Trigonometric and Hyperbolic Identities

Trigonometric	**Hiperbolic**
$\cos^2 x + \sin^2 x = 1$	$\sinh x = \dfrac{e^x - e^{-x}}{2}$
$1 + \tan^2 x = \sec^2 x$	$\cosh x = \dfrac{e^x + e^{-x}}{2}$
$\cot^2 x + 1 = \mathrm{cosec}^2 x$	$\cosh^2 x - \sinh^2 x = 1$
$\sin 2x = 2\sin x \cos x$	$1 - \tanh^2 x = \mathrm{sech}^2 x$
$\cos 2x = \cos^2 x - \sin^2 x$	$\coth^2 x - 1 = \mathrm{cosech}^2 x$
$\cos 2x = 2\cos^2 x - 1$	$\sinh 2x = 2\sinh x \cosh x$
$\cos 2x = 1 - 2\sin^2 x$	$\cosh 2x = \cosh^2 x + \sinh^2 x$
$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$	$\cosh 2x = 2\cosh^2 x - 1$
$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$	$\cosh 2x = 1 + 2\sinh^2 x$

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$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$	$\tanh 2x = \dfrac{2\tanh x}{1+\tanh^2 x}$
$\tan(x \pm y) = \dfrac{\tan x \pm \tan y}{1 \mp \tan x \tan y}$	$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$
$2\sin x \cos y = \sin(x+y) + \sin(x-y)$	$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$
$2\sin x \sin y = -\cos(x+y) + \cos(x-y)$	$\tanh(x \pm y) = \dfrac{\tanh x \pm \tanh y}{1 \pm \tanh x \tanh y}$
$2\cos x \cos y = \cos(x+y) + \cos(x-y)$	

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