UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	NUMERICAL METHOD
COURSE CODE	:	BFC 25203
PROGRAMME CODE	:	BFF
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTIONS	:	1. ANSWER **ALL** QUESTIONS.
		2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **CLOSE BOOK**.

THIS QUESTION PAPER CONSISTS OF **EIGHT (8)** PAGES

Q1 (a) The population of a small town was recorded in a census system for every ten (10) years. **Table Q1 (a)** shows that the population has a positive trend.

(i) Approximate the population during 1966.

(9 marks)

(ii) Based on the **Q1 (a)(i)**, suggest the method that could potentially provide the best approximation. Give a justification for your answer.

(2 marks)

(b) A projectile is thrown into space. A speed gauge equipped with a camera is prepared to capture its trajectory every **TWO (2)** seconds, with $0 \leq t \leq 4$. The speed increased from 1 m/s to 2 m/s before it stopped.

(i) Interpolate the cubic spline of the trajectory.

(9 marks)

(ii) Construct its cubic polynomial equation

(5 marks)

Q2 (a) **Table Q2 (a)** gives the values of distance travelled by a car at various times from a traffic light. Given that velocity, $v = x'(t)$ and acceleration, $a = v'(t)$.

(i) Determine approximate **SIX (6)** values of the car's velocity at t = 7.5 min using appropriate difference formula and a consistent step size. Answers must be in 3 decimal places.

(10 marks)

(ii) Based on the **Q2 (a)(i)**, suggest the method that could potentially provide the best approximation. Give a justification for your answer.

(2 marks)

(iii) Estimate the approximate acceleration of the car (in 3 decimal places) at a distance of 12 km by considering as much data as possible.

(3 marks)

(b) The velocity of a particle which starts from rest is given by **Table Q2 (b)**:

(i) Examine the approximate total distance travelled in 20 seconds using a suitable Simpson's rule.

(8 marks)

(ii) Explain the reason of chosen rule in **Q2(b)(i)**.

(2 marks)

Q3 The vibration modes and the motion pattern of a bridge system can be foreseen through the application of eigenvalue and eigenvector. The dynamic system of the bridge can be expressed in the matrix form as:

$$A = \begin{bmatrix} a & 1 & 3 \\ 1 & 0 & 1 \\ 2 & 1 & a \end{bmatrix}$$

where a is the last digit of your matrix number. If the last digit of your number is zero or one then take $a = 2$. Use try value $\mathbf{v}^{(0)} = [1 \quad 0 \quad 1]^T$ and calculate until $|m_{k+1} - m_k| \leq 0.005$ or **FIVE (5)** iterations whichever comes first. Do the calculations in 3 decimal places.

(a) Estimate the mode shape of the vibration by finding the dominant (in absolute value) eigenvalue and its motion pattern (corresponding eigenvector).

(11 marks)

(b) Determine the smallest (in absolute) eigenvalue and corresponding eigenvector using shifted power method.

(14 marks)

Q4 (a) The falling parachutist satisfies the following differential equation:

$$\frac{dv}{dt} = g - \frac{c}{m}v,$$

where v is the velocity of the parachutist (m/s), t is time (s), g is gravity acceleration (m/s^2), c is drag coefficient (kg/s) and m is the mass of the parachutist. Take $g = 9.8067$, m as your own weight and c is the last digit of your matrix number, If the last digit of your number is zero then take c = 10. Estimate the velocity of the parachutist till time $t = 2$ using the Euler's and fourth-order Runge-Kutta method with $\Delta t = 1$ and $v0 = 0$. Find exact solution then find the absolute errors for each method. Conclude which method is more accurate.

(13 marks)

(b) A test was conducted to determine the temperature distribution of a one meter iron reinforcement bar under fire. The initial temperature of the reinforcement bar is given by $U(x, 0) = Sin\ 2\pi\chi$ for $0 \leq \chi \leq 1$. The boundary conditions $U(0, t) = U(1, t) = 0$ for $0 \leq \chi \leq 0.01$. Given the heat equation is $\frac{\partial u}{\partial t} = \frac{\partial u^2}{\partial x^2}$ by taking $\Delta x = 0.25$ and $\Delta t = 0.01$ and $C2 = 0.23$:

(i) Examine the system of linear equation of the temperature distribution by using the most stable finite difference method.

(10 marks)

(ii) Justify why the chosen finite difference method in **Q4 (a)(i)** is the most stable method.

(2 marks)

- END OF QUESTIONS -

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Table Q1(a): The record of town population over ten years.

Year	1951	1961	1971
Population (thousand)	2.8	3.2	4.5

Table Q2(a): Distance as a function of time.

$Time, t\ (min)$	$Distance, x\ (km)$
3.0	4.600
4.0	6.271
4.5	7.055
5.0	8.030
5.5	8.518
6.5	10.996
7.0	12.000
7.5	13.222
8.5	15.007
9.0	16.285
9.5	17.200
10.0	18.096
11.00	19.904

Table Q2(b): Speed every two seconds epoch.

$t\ (s)$	0	2	4	6	8	10	12	14	16	18	20
$v\ (ft/s)$	0	16	29	40	46	51	32	18	8	3	0

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FORMULAE

INTERPOLATION

Lagrange

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f_i, \qquad L_i = \prod_{\substack{j=0 \\ j\neq i}}^{n} \frac{(x-x_j)}{(x_i-x_j)}$$

Natural Cubic Spline Polynomial

$$h_k = x_{k+1} - x_k\,, \quad k = 0,1,2,\cdots,n-1$$

$$d_k = \frac{f_{k+1}-f_k}{h_k}, \quad k = 0,1,2,\cdots,n-1$$

$$b_k = 6(d_{k+1}-d_k), \quad k = 0,1,2,\cdots,n-2$$

$$h_k m_k + 2(h_k + h_{k+1})m_{k+1} + h_{k+1}m_{k+2} = b_k, \quad k = 0,1,2,\cdots,n-2$$

$$S_k(x) = \frac{m_k}{6h_k}(x_{k+1}-x)^3 + \frac{m_{k+1}}{6h_k}(x-x_k)^3 + \left(\frac{f_k}{h_k} - \frac{m_k}{6}h_k\right)(x_{k+1}-x) + \left(\frac{f_{k+1}}{h_k} - \frac{m_{k+1}}{6}h_k\right)(x-x_k) \quad \text{where } k = 0,1,2,\cdots,n-1$$

DIFFERENTIATION

2-point forward

$$f'(x) \approx \frac{f(x+h)-f(x)}{h}$$

2-point backward

$$f'(x) \approx \frac{f(x)-f(x-h)}{h}$$

3-point central

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

3-point forward

$$f'(x) \approx \frac{-3f(x)+4f(x+h)-f(x+2h)}{2h}$$

3-point backward

$$f'(x) \approx \frac{f(x-2h)-4f(x-h)+3f(x)}{2h}$$

5-point central

$$f'(x) \approx \frac{f(x-2h)-8f(x-h)+8f(x+h)-f(x+2h)}{12h}$$

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FORMULAE

DIFFERENTIATION

3-point central

$$f''(x) \approx \frac{f(x-h) - 2f(x) + f(x+h)}{h^2}$$

5-point central

$$f''(x) \approx \frac{-f(x-2h) + 16f(x-h) - 30f(x) + 16f(x+h) - f(x+2h)}{12h^2}$$

INTEGRATION

Simpson's $\frac{1}{3}$ Rule

$$A_1 = \int_{x_0}^{x_2} f(x)dx \approx \frac{h}{3}(f_0 + 4f_1 + f_2)$$

$$\int_{a=x_0}^{b=x_n} f(x)dx \approx \frac{h}{3}\left[f_0 + f_n + 4\sum_{i=1}^{n/2} f_{2i-1} + 2\sum_{i=1}^{\left(\frac{n}{2}\right)-1} f_{2i}\right]$$

Simpson's $\frac{3}{8}$ Rule

$$A_1 = \int_{x_0}^{x_3} f(x)dx \approx \frac{3}{8}h(f_0 + 3f_1 + 3f_2 + f_3)$$

$$\int_{a=x_0}^{b=x_n} f(x)dx \approx \frac{3}{8}h\left[f_0 + f_n + 3\sum_{i=1}^{n/3}(f_{3i-2} + f_{3i-1}) + 2\sum_{i=1}^{\left(\frac{n}{2}\right)-1} f_{3i}\right]$$

EIGENVALUES

Power method

$$v^{(k+1)} = \frac{1}{m_{k+1}} A v^{(k)}, \qquad k = 0,1,2,\cdots$$

Shifted Power Method

$$v^{(k+1)} = \frac{1}{m_{k+1}} A_{shifted} v^{(k)}, \qquad k = 0,1,2,\cdots$$

$$A_{shifted} = A - \lambda_1 I$$

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FORMULAE

ODEs

Euler's Method

$$y_{i+1} = y_i + hy_i' = hf(x_i, y_i)$$

Fourth-Order Runge-Kutta Method (RK4)

$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4),$$

$$k_1 = hf(x_i, y_i),$$

$$k_2 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_1}{2}\right),$$

$$k_3 = hf\left(x_i + \frac{h}{2}, y_i + \frac{k_2}{2}\right),$$

$$k_4 = hf(x_i + h, y_i + k_3),$$

PDEs

Heat Equation: Explicit Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j}$$

$$\frac{u_{i,j+1} - u_{i,j}}{k} = c^2 \left(\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$

Heat Equation: Implicit Crank-Nicolson Method

$$\left(\frac{\partial u}{\partial t}\right)_{i,j} = \left(c^2 \frac{\partial^2 u}{\partial x^2}\right)_{i,j}$$

$$\frac{u_{i,j+1} - u_{i,j}}{k} = \frac{c^2}{2}\left(\frac{u_{i-1,j+1} - 2u_{i,j+1} + u_{i+1,j+1}}{h^2} + \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j}}{h^2}\right)$$