Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	COMPUTATIONAL METHOD
COURSE CODE	:	MDC 11104
PROGRAMME CODE	:	MDM
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1. ANSWER **FIVE (5)** QUESTIONS **ONLY** 2. THIS FINAL EXAMINATION IS AN **ONLINE** ASSESSMENT AND CONDUCTED VIA **OPEN BOOK** AND YOU MAY USE ONLY **ONE (1)** TEXT BOOK.

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

Q1 A parachutist of mass $\boldsymbol{m} = 75$ kg jumps out of a hovering helicopter. The following expression models the acceleration of falling objects:

$$\frac{dv}{dt} = g - \frac{cv^2}{m}$$

In this equation, $\boldsymbol{dv / dt}$ is the acceleration, $\boldsymbol{g}$ is the gravitational acceleration at 9.81 m/s^2, and $\boldsymbol{c}$ is the drag coefficient equal to 0.25 kg/m.

(a) Assuming that the initial velocity is zero, find the approximate velocities of the parachutist at $\boldsymbol{t} = 3, 6, 9$, and 12 seconds using Euler's method.

(15 marks)

(b) The terminal velocity of the parachutist, v_∞, can be calculated by setting the acceleration of the previous equation to zero:

$$v_\infty = \sqrt{\frac{mg}{c}}$$

Compare the velocity value from this equation with your answer at $\boldsymbol{t} = 12$ in Q1(a). How can we improve upon the approximations made by Euler's method?

(5 marks)

Q2 The specific volume of a gas (v) can be expressed as a function of its pressure p and temperature T, using van der Waals equation:

$$p = \frac{KT}{v - b} - \frac{a}{v^2}$$

In this equation, $K = 0.518$ kJ / (kg K) is the gas constant, whilst a = 0.889 kN m^4 / kg^2 and b = 2.68 x 10^3 m^3 / kg are constants.

(a) Find the approximate specific volume of methane gas at a pressure of 100 kPa and temperature 255.4 K using a graphical approach.

(8 marks)

(b) Evaluate the solution of Q2(a) using Newton-Raphson method with the starting value of $v = 2681.0$ and $\varepsilon = 0.1$.

(8 marks)

(c) Compare the two methods using the help of plots. Briefly comment on the advantages of each method.

(4 marks)

Q3 (a) Given the differential equation with the initial condition y (0) = 0 as

$$\frac{\partial y}{\partial x} = e^{-x} - y$$

Estimate the value of y (0.2) using fourth order Runge-Kutta method with h =0.1 and compare your results with the analytical solution $y(x) = xe^{-x}$.

(16 marks)

(b) Using 2nd order central difference, write the finite difference equation for the heat transfer equation:

$$\frac{\partial T}{\partial t} = a(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2})$$

(i) In explicit form.

(ii) In implicit form.

(4 marks)

Q4 Consider a square plate $1.5m \times 1.5m$ that is subjected to the boundary conditions shown in **Figure Q4**. The equation that governs the steady state heat transfer is the Laplace equation;

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

(a) Discretize the above elliptic partial differential equation and form the finite difference equation.

(8 marks)

(b) Evaluate the temperature at the interior nodes using a square grid with a length of 0.5 m by using Gauss-Siedel iteration method. Assume the initial temperature at all interior nodes to be 0°C. Use 2nd order central difference and **stop after 2 iterations.**

(12 marks)

Q5 A slender column with length $L = 4$ m is subject to a load F. This system can be modelled as

$$\frac{d^2 y}{dx^2} + p^2 y = 0$$

where $F = p^2 EI$ and its boundary is given as $x(0) = x(4) = 0.0$.

(a) Approximate the model using first derivatives central finite different method by dividing it into 4 sections. Formulate the system of equations in matrix form.

(5 marks)

(b) Find all possible value of p^2 by polynomial method and evaluate the values using Gerschogorin's Theorem.

(9 marks)

(c) Determine the corresponding eigenvectors based on all possible value of p^2 found in **Q5(b)**.

(6 marks)

Q6 The matrix [A] is given as:

$$[A] = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

The Power Method represents one of method can be used to determine the largest of the eigenvalue for any given matrix.

(a) Describe the basic idea of Power Method.

(3 marks)

(b) If the given initial value of its eigenvector is $x^{(0)} = [1,1,1]^T$, find the first four iterations result of the Power method.

(5 marks)

(c) Reduce matrix [A] into upper triangular matrix using QR decomposition with Householder Transformation method.

(8 marks)

(d) Explain how to find all eigenvalues using QR algorithm.

(4 marks)

- END OF QUESTIONS -

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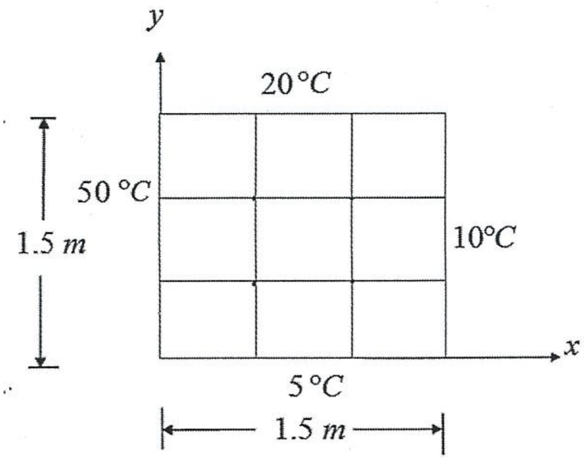

Figure Q4: The boundary condition for square plate