Universiti Tun Hussein Onn Malaysia

UNIVERSITI TUN HUSSEIN ONN MALAYSIA

FINAL EXAMINATION
SEMESTER I
SESSION 2021/2022

COURSE NAME	:	DISCRETE STRUCTURE
COURSE CODE	:	BIT 11003
PROGRAMME CODE	:	BIT
EXAMINATION DATE	:	JANUARY / FEBRUARY 2022
DURATION	:	3 HOURS
INSTRUCTION	:	1.ANSWER **ALL** QUESTIONS. 2.THIS FINAL EXAMINATION IS CONDUCTED ONLINE AND CLOSE BOOK. 3.THE STUDENTS SHOULD UPLOAD THE ANSWER BOOKLET (PDF FORMAT) WITHIN 30 MINUTES AFTER EXAMINATION PERIOD.

THIS QUESTION PAPER CONSISTS OF **FIVE (5)** PAGES

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Q1 (a) Given $\alpha = (a + b' + c') \cdot (a + b' + c)$ and $\beta = a + b' \cdot c$. Show whether $\propto = \beta$ by using the truth table.

(5 marks)

(b) A combinational logic circuit that has the truth table outputs as in **TABLE 1**.

TABLE 1: Truth Table of Logic Circuit

x	y	z	S_1
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

a	b	c	S_2
0	0	0	0
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	1
1	0	1	1
0	1	1	0
1	1	1	1

Obtain

(i) Conjuctive Normal Form (CNF) for S_1.

(3 marks)

(ii) Disjuctive Normal Form (DNF) for S_2.

(3 marks)

(c) Let $P(x, y)$: $2x + y = 1$, where the universe of discourse is the set of all integers. Examine the truth values of the following:

(i) $\forall x \exists y P(x, y)$

(2 marks)

(ii) $\forall x \forall y P(x, y)$

(2 marks)

(d) At the end of each year, students at EZ School received ribbons at a school-wide awards ceremony. This year, 120 students receive gold stars for perfect attendance, 180 receive certificates for participating in the science fair and 80 students receive blue ribbons for outstanding grades. Of these, 40 students who receive the attendance star receive no other award, 50 students who receive the science fair certificate receive no other award, and 10 students receive the blue ribbon receive no other award. In addition, 10 students receive all three awards and 65 students receive no awards.

(i) Illustrate this situation by a Venn diagram.

(2 marks)

(ii) Compute how many students attend the school this year.

(3 marks)

Q2 (a) Given $X = \{1,3,5\}$. R_L is a relation on X, defined by aR_Lb *if and only if* $|a - b|$ is an even integer.

(i) List all possible elements of set R_L .

(3 marks)

(ii) Write the matrix represent R_L .

(2 marks)

(b) Let $B = \{r, t, s\}$ and R_2 is a relation on set defined by the following matrix

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(i) Draw the digraph for the relation R_2.

(2 marks)

(ii) Examine all possible properties of R_2. It is reflexive, symmetric or transitive? Justify your answer.

(6 marks)

(c) Let $V=\{1,2,3,4\}$ and $W=\{a,b,c\}$.
Function $f: V \rightarrow W$ is defined as follows:

$f(1) = c, f(2) = a, f(3) = c, f(4) = b.$

Function $h: V \rightarrow W$ is defined as follows:

$h(1) = c, h(2) = b, h(3) = b, h(4) = c.$

Analyze which of these functions is onto? Justify your answer.

(4 marks)

(d) If $g(x) = \sqrt{x+1}$, derive $g^{-1}(y)$.

(4 marks)

(e) Given the function $f(x) = 2x + 3$, $g(x) = -x^2 + 1$ and $h(x) = 1/x$, $x \neq 0$. Formulate the composition of $h \circ (g \circ f)$.

(5 marks)

Q3 (a) Using the mathematical induction, verify that the following statement is true for all non-negative integers n.

$$1 \cdot (1!) + 2 \cdot (2!) + \cdots + n \cdot (n!) = (n+1)! - 1$$

(6 marks)

(b) Compute s_5 if $s_0, s_1, s_2, \ldots$ is a sequence satisfying the given recurrence relation and initial conditions.

$$s_n = 2s_{n-1} + s_{n-2} - s_{n-3} \quad \text{for } n \geq 3, s_0 = 2, \; s_1 = -1, s_2 = 4.$$

(3 marks)

(c) Determine the solution of second-order linear homogeneous recurrence relation

$$a_n = 2a_{n-1} + 15a_{n-2}$$

with $a_0 = 1$ and $a_1 = -4$.

(6 marks)

Q4 (a) Solve for x if $25x \equiv 15 (\mathrm{mod}\ 29)$.

(5 marks)

(b) Determine the smallest positive integer so that when divided by 3, 5 and 7, the reminder is 1, 4, 6 respectively.

(9 marks)

Q5 (a) A short road race has been organized by FSKTM's ICT club for students and staffs. Participants have to start at point A and use their own choice of route to reach endpoint J as quickly as possible. **Figure Q5(a) as Graph Z** shows the network of roads available and the minimum completion time (in seconds) for each road.

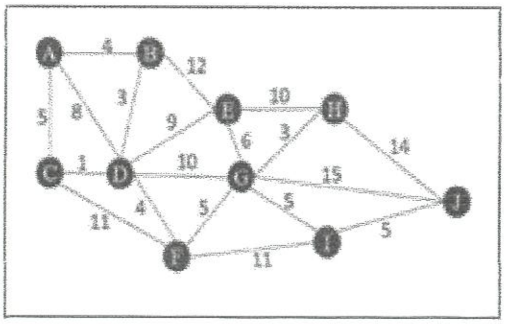


figure Q5(a): Graph Z

(i) State the degree for each vertex in Graph Z.

(5 marks)

(ii) Represent Graph Z by

(a) the incidence Matrix.

(2 marks)

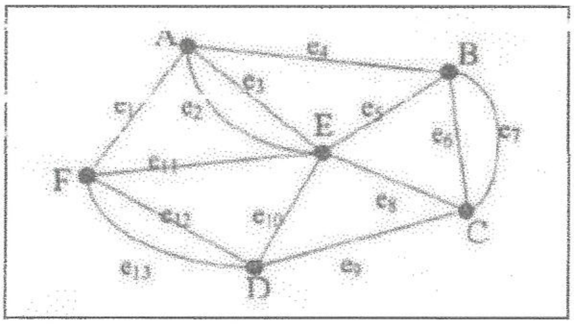
(b) the Adjacency List.

(2 marks)

(iii) Mira, the best runner is planning her strategy. Given that she can run each section in the minimum time, use Dijkstra's algorithm to determine the shortest route she should take. State her time for the race and draw the route. Show all your iteration in a table.

(9 marks)

(b) Consider the following Figure Q5(b) as **Graph *D***.



Graph D

(i) Determine whether there exists Euler Cycle or Euler Path? If any, exhibit only one.

(4 marks)

(ii) Analyze whether Graph D has Hamiltonian Cycle? Justify your answer.

(3 marks)

- END OF QUESTIONS -